Oct. 1962 76p. 37 ref

N63 23022

CODE-1

(NASA TMX 50841)

OTS PRICE

XEROX $ 7.60 ph

MICROFILM $ 2.48 mf

[6]

National A - - - - - - - .

GODDARD SPACE FLIGHT CENTER, Greenbelt, Md.

[redacted]

IONOSPHERIC ABSORPTION OF EXTRATERRESTRIAL RADIO WAVES

; I-615-62-194)

October 1962

by

Larry W. Brown
Planetary Ionospheres Branch
Space Sciences Division

IONOSPHERIC ABSORPTION OF

EXTRATERRESTRIAL RADIO WAVES

by

Larry W. Brown
National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

ABSTRACT

23022 (oae)

The theory of ionospheric absorption of extraterrestrial radio waves is reexamined as a means of investigating the properties of the ionosphere. The absorption is shown to be an integration of the real part of the complex ionospheric conductivity over the ray path. The calculation of the conductivity tensor differs from the classical Appleton-Hartree formulization in that the electron collision frequency is dependent on the velocity of the particles.

A model ionosphere is constructed from recent rocket and satellite observations. This model shows that under equilibrium conditions only electron interactions with ions and collisions with neutral nitrogen and oxygen molecules contribute significantly to the absorption. A separate calculation of the absorption due to electron-neutral collisions, and that due to electron-ion interactions shows that each phenomenon occurs in a distinctive region of the ionosphere. Electron-neutral collisions dominate the D region of the ionosphere while electron-ion interactions are predominate in

23022

the F region. The total absorption measured by a ground-based antenna is the sum of the absorption in each of these regions as the contribution from other regions is negligible. The total absorption of a normal day ionosphere is found to be 0.779 db. while for a normal night ionosphere the absorption is 0.005 db. for a receiving frequency of 24.3 Mc/sec. Clearly then, absorption can be measured by comparison of the different powers received at the same sidereal time. This validates the cosmic radio noise method of measuring ionospheric absorption. Although the absorption takes place in different parts of the ionosphere, no quantitative method of separation is found.

The principle of riometer measurements of extraterrestrial radio noise is reexamined in order to determine the precautions necessary to insure that only the cosmic noise power with absorption is measured. Methods are presented for reduction of the riometer records in terms of the absorption experienced by the radio waves at particular times. Author

CONTENTS

LIST OF FIGURES

I. INTRODUCTION

The first discovery of cosmic radio waves was made in 1932 by Jansky[1] while studying atmospheric static. It was found that our Galaxy is a source of radio frequency emission with maximum intensity in the direction of the galactic center. Due to ionospheric absorption the cosmic radio wave intensity was found to be lower than expected during the day[2].

Thus, the effect of the ionosphere on cosmic radio noise can be measured by comparing the power actually received by a fixed receiving system with the power received on the same system at the same sidereal time under conditions of negligible ionospheric absorption[3]. Observations of 18.3 Mc/sec cosmic noise have confirmed that although ionospheric absorption is small, it can be measured without difficulty[4].

The absorption of extraterrestrial radio waves is used extensively to study the ionosphere by means of the riometer (relative ionospheric opacity meter). Interpretation of its measurements leads to an identification of ionospheric disturbances, during which the absorption of cosmic noise is enhanced. Analysis of the variation of absorption with time during such events can be utilized in correlations with data from rocket probes and satellites such as orbiting solar observatories and ionospheric sounders to obtain properties of the ionosphere.

The effects of solar activity on the different regions of the ionosphere such as flare activity on the F_2 region and the disturbance of the D region due to increased solar radiation influx are readily investigated using riometer absorption measurements. Since the ionospheric effects of solar activity are confined mainly to the D region, the riometer can also serve as a passive ground-based instrument for the support of D region rocket experiments.

The riometer is a simple radio telescope operating at low frequencies where such radio observation of the sky is seriously hampered by the ionosphere. As such its use serves as an introduction to the important new field of low-frequency radio astronomy. With a capability of allowing for ionospheric effects, it will be possible to study the extended atmospheres of the sun, moon, and planets; support radio mapping of the sky by orbiting radio observatories with ground-base observations; and extend the integrated power spectrum of cosmic radio sources.

It is the purpose of this report to examine critically the theory of ionospheric absorption in order to verify the cosmic radio noise method[5] of analysis and its various applications to the ionosphere and radio astronomy.

II. BASIC THEORETICAL ANALYSIS OF IONOSPHERIC ABSORPTION

The Appleton-Hartree magneto-ionic theory[6] presented in

1932 shows that radio frequency waves traveling in an ionized medium will be attenuated. The absorption is due to collisions between oscillating free electrons and the other particles in the medium. The free electrons oscillate at the frequency of the incident radio wave, and then through collisions with other particles lose part of their oscillatory energy gained from the radio field.

The absorption may be written as follows[7]:

$$D = -10 \log \frac{P}{P_o} \tag{1}$$

where, D is the ionospheric absorption in decibels, P is the actual received power of the cosmic waves, P_o is the received power of the cosmic waves on the same receiving system at the same sidereal time as P except under conditions of negligible ionospheric absorption.

Consider now the total received energy of an electromagnetic wave per unit solid angle[8].

$$\frac{dW}{d\Omega} = \int_0^{\infty} \frac{dI(\omega)}{d\Omega} \, d\omega$$

Then, I is the intensity, i.e. energy radiated per unit solid angle per unit frequency interval, and equation (1) may be written:

$$D = -10 \log \frac{I}{I_o} \tag{2}$$

where, I_o is the intensity at the source and $\frac{I}{I_o} = \frac{P}{P_o}$.

If one assumes that the electromagnetic radiation is absorbed in the ionosphere according to Lambert's Law[8], then

$$\frac{dI}{I} = -\alpha(s)ds, \tag{3}$$

where, the fraction of intensity lost $\frac{dI}{I}$ in tranversing an infinitesimal thickness ds is proportional to ds, and the proportionality factor is a function of the distance along the path.

Integrating equation (3) yields

$$I = I_o \exp\left(-\int_0^\infty \alpha(s)ds\right) \tag{4}$$

where, $\int_0^\infty \alpha(s)ds$ is an integration of the attenuation over the entire path length. However, the intensity is proportional to the square of the electric field strength. Therefore, rewriting equation (4)

$$E = E_o \exp\left(-\int_0^\infty x\, ds\right) \tag{5}$$

where, E_o is the electric field strength at the source, and x is the attenuation of the electric field and thus is one half the attenuation (α) of the intensity.

Equation (2) may be written in terms of the electric field strength.

$$D = -20 \log \frac{E}{E_o} \tag{6}$$

Substituting equation (5),

$$D = -20 \log \exp\left(-\int_0^\infty x\, ds\right) \tag{7a}$$

or,

$$D = 8.69 \int_0^{\infty} x \, ds. \tag{7b}$$

From electromagnetic field theory, the electric field in vacuum may be represented as a wave[8].

$$\tilde{E} = \tilde{A} \, e^{-(j\omega t - ka)}, \tag{8}$$

where, $\tilde{A}$ is the amplitude of the electromagnetic wave,

w is the angular frequency of the electromagnetic wave,

t is the time,

k is the propagation number ($2\pi/\lambda$) in vacuum,

a is the distance from the source.

If this wave is traveling in a medium which absorbs part of the electromagnetic wave, then the propagation number suffers an attenuation such that

$$x = \frac{2\pi\kappa}{\lambda}, \tag{9}$$

or, in terms of the frequency

$$x = \frac{\omega\kappa(h)}{c} \tag{10}$$

where, κ is the attenuation coefficient. A wave traveling through the ionosphere encounters an absorption which varies with atmospheric properties such as temperature, electron density, and molecular density gradients; therefore, the attenuation coefficient is a function of the height distribution of these parameters.

A calculation of the attenuation factor must proceed from the macroscopic electromagnetic properties of an ionized gas which are specified by the complex conductivity[9]. The macroscopic description proceeds from the Maxwellian equation.

$$\tilde{\nabla} \times \tilde{H} = 4\pi\tilde{J} + \frac{d\tilde{E}}{dt} \tag{11}$$

Where, $\tilde{H}$ is the magnetic field,

$\tilde{J}$ is the current density.

In terms of a dielectric constant (K) equation (11) becomes

$$\tilde{\nabla} \times \tilde{H} = K \frac{d\tilde{E}}{dt} . \tag{12}$$

The time dependent part of the electric field may be written as

$$\tilde{E} = \tilde{A} \, e^{-j\omega t}, \tag{13}$$

and the current density ($\tilde{J}$) is related to the electric field

$$\tilde{J} = \sigma\tilde{E}, \tag{14}$$

where σ is the complex conductivity.

$$\sigma = \sigma_r + j\sigma_i, \tag{15}$$

with σ_r = real part of the conductivity,

σ_i = imaginary part of the conductivity.

Combining equations (11) and (12),

$$K \frac{d\tilde{E}}{dt} = 4\pi\tilde{J} + \frac{d\tilde{E}}{dt}, \tag{16}$$

and substituting equations (13), (14), and (15),

$$K \frac{d}{dt} (\tilde{A} \, e^{-j\omega t}) = 4\pi(\sigma_r + j\sigma_i) \tilde{A} \, e^{-j\omega t} + \frac{d}{dt} (\tilde{A} \, e^{-j\omega t}). \tag{17}$$

Equation (17) reduces to

$$-j\omega K = 4\pi(\sigma_r + j\sigma_i) - j\omega, \tag{18}$$

or,

$$K = 1 + j \frac{4\pi}{\omega} (\sigma_r + j\sigma_i) \quad . \tag{19}$$

The dielectric constant is related to the refractive index (μ) and the attenuation coefficient (χ) from the relation

$$K = (\mu - j\chi)^2 . \tag{20}$$

Substituting in equation (20) and separating the real and imaginary parts of the equation yields

$$\mu^2 - \chi^2 = 1 - \frac{4\pi}{\omega}\sigma_i , \tag{21a}$$

$$\mu\chi = \frac{2\pi}{\omega}\sigma_r . \tag{21b}$$

For the ionosphere the complex conductivity ($\sigma_r + j\sigma_i$) in equation (19) is small compared to one; therefore, expansions of the refractive index and the attenuation coefficient may be made neglecting second order and higher terms of the complex conductivity.

From equation (21b)

$$\chi^2 = \left(\frac{2\pi}{\omega\mu}\right)^2 \sigma_r^2 \tag{22}$$

χ^2 is a second order term in the complex conductivity, and therefore, can be neglected in equation (21a).

$$\mu = \left(1 - \frac{4\pi}{\omega}\sigma_i\right)^{\frac{1}{2}} \tag{23}$$

Expansion of the square root yields

$$\mu = 1 - \frac{2\pi}{\omega}\sigma_i \quad . \tag{24}$$

Substituting equation (24) in equation (21b),

$$\chi = \frac{2\pi}{\omega}\sigma_r \left(1 - \frac{2\pi}{\omega}\sigma_i\right)^{-1} . \tag{25}$$

Expanding equation (25) and neglecting products of the real and imaginary parts of the complex conductivity as second order terms, yields

$$\mathcal{X} = \frac{2\pi}{\omega}\sigma_{real}. \tag{26}$$

If equations (26) and (10) are substituted in equation (7b), the calculation of the ionospheric absorption reduces to a calculation of the real part of the complex conductivity of the ionosphere.

$$D = 8.69 \int_0^\infty \frac{2\pi}{c}\,\sigma_{real}\;ds. \tag{27}$$

III. <u>CONDUCTIVITY OF THE IONOSPHERE</u>

One of the major differences between observed and calculated values of conductivity of the normal ionosphere lies in the limitation of the Appleton-Hartree formulization of the physical theory(10).

The limitations of the theory arise from a simplified treatment of the electron collisions with other particles(11). (i.e. collisions independent of velocity). A more exact derivation of the conductivity of an ionized medium proceeds from the application of Boltzmann's equation(12).

$$\frac{\partial f}{\partial t} + \tilde{\nabla}_r \cdot \tilde{\gamma} + \tilde{\nabla}_v \cdot \tilde{g} = B(t), \tag{28}$$

and,

$$\tilde{\gamma} = \tilde{v} f, \tag{28a}$$

$$\tilde{g} = (\tilde{a} + \tilde{\omega}_b \times \tilde{V})f, \qquad (28b)$$

where, f is the electron distribution function,

$\tilde{\nabla}_r$ is the gradient with respect to position,

$\tilde{\nabla}_v$ is the gradient with respect to velocity(v),

$\tilde{a}$ is the acceleration due to external forces,

$\tilde{\omega}_b$ is the gyromagnetic frequency $(\frac{eH}{mc})$,

B(t) is the interaction of electrons with all particles in the ionosphere in impact velocity space.

The electron interactions can be represented as follows[13]:

$$B(t) = \left(\frac{\delta f}{\delta t}\right)_{en} + \left(\frac{\delta f}{\delta t}\right)_{ei}, \qquad (29)$$

where, $\left(\frac{\delta f}{\delta t}\right)_{en}$ is the collision interaction due to electrons and neutral particles,

$\left(\frac{\delta f}{\delta t}\right)_{ei}$ is the collision interaction due to electrons and ions, and electron-electron interactions are neglected.

In order to solve the Boltzman equation in a simplified form for conditions of the ionosphere, certain basic assumptions must be made[13].

(1) The basic equations result from an application of the Fokker-Planck equation for coulomb interaction andthe Boltzman collision equation for electron-neutral collision.

(2) Only the electron current and heat flow are considered.

(3) Moderate temperatures exist so that the a-c charge density may be neglected.

(4) Only elastic collisions are considered.

(5) A steady state is assumed to exist.

(6) The collision frequency is a function of velocity only; that is, the scattering centers are isotropically distributed in space.

The electron distribution function is in general a Fourier series expansion in ωt[5].

$$f = \sum_{\ell} \sum_{m} f_m^{\ell} P_{\ell} \; e^{jm\omega t}, \tag{30}$$

where, $f_o^{\,o}$ is the dc electron distribution function,

$f_1^{\,o}$ is the dc current distribution function,

$f_o^{\,1}$ is the ac electron distribution function (assumed zero),

$f_1^{\,1}$ is the ac current distribution function.

and,

$$\frac{\partial f}{\partial t} = \sum_{\ell} \sum_{m} jm\omega \, f_m^{\ell} \, P_{\ell} \; e^{jm\omega t}. \tag{31}$$

These exponential notations are convenient for linear equations; however, care must be used in evaluating products such as $\tilde{a}f^{\ell}$.

$$\tilde{a}f^{\ell} = \frac{\tilde{a}}{2}\Big[f^{\ell}_{1_{real}} + (2f_o^{\ell} + f_2^{\ell})e^{j\omega t} + \\ + \sum_{m=2}^{\infty}(f^{\ell}_{m-1} + f^{\ell}_{m+1})e^{jm\omega t}\Big] \tag{32}$$

Since $\tilde{\nabla}_r \cdot \tilde{f}_o^{o}$ and $\tilde{\nabla}_v(\tilde{\omega}_b \times \tilde{v}) = 0$, equation (28) may be written in the following form:

$$\frac{\partial f}{\partial t} + (\tilde{\nabla}_r \cdot \tilde{v} f) + \tilde{\nabla}_v(\tilde{a}f) \\ + \tilde{\nabla}_v \cdot (\tilde{\omega}_b \times \tilde{v}f) = B(t). \tag{33}$$

The use of equations (30), (31), and (32) in equation (33) yields the following result with a Legendre expansion in the magnetic field term[12].

$$\tilde{B}_o^o = \frac{v}{3}\tilde{\nabla}_r \cdot \tilde{f}_1^o + \frac{1}{3v^2}\frac{d}{dv}\left(\frac{v^2}{2}\tilde{a}\cdot\tilde{f}_{1\,real}^1\right), \tag{33a}$$

$$\tilde{B}_1^o = v\,\tilde{\nabla}_r f_o^o - \tilde{\omega}_b \times \tilde{f}_1^o, \tag{33b}$$

$$\tilde{B}_1^1 = j\omega f_1^1 + \tilde{a}^1\frac{d}{dv}f_o^o - \tilde{\omega}_b \times \tilde{f}_1^1, \tag{33c}$$

with $\tilde{a} = \tilde{a}^o + \tilde{a}^1\ e^{j\omega t}$,

or, $\tilde{a} = \frac{e\tilde{E}^o}{m} + \frac{e}{m}\tilde{E}^1\ e^{j\omega t}$,

where, ω is the angular radio frequency,

$\tilde{E}^o$ is the dc electric field,

$\tilde{E}^1$ is the ac electric field,

e is the electronic charge,

m is the electronic mass.

Applying the Boltzman collision equation for electron-neutral interaction and the Fokker-Planck equation for coulomb collisions to equation (29)[13].

$$B_o^o = \left(\frac{\delta f_o^o}{\delta t}\right)_{en} + \left(\frac{\delta f_o^o}{\delta t}\right)_{ei}, \tag{34a}$$

with,

$$\left(\frac{\delta f_o^o}{\delta t}\right)_{en} = \frac{1}{v^2}\frac{\partial}{\delta v}\left[v^3\,\nu_{en}\,\frac{m}{M}\left(f_o^o + \frac{kTg}{mv}\,\frac{f_o^o}{v}\right)\right],$$

$$\left(\frac{\delta f_o^o}{\delta t}\right)_{ei} = \frac{1}{v^2}\frac{\partial}{\delta v}\left[v^3\,\nu_{ei}\,\frac{m}{M}\left(f_o^o + \frac{kTg}{mv}\,\frac{f_o^o}{v}\right)\right].$$

$$B_1^o = \left(\frac{\delta f_1^o}{\delta t}\right)_{en} + \left(\frac{\delta f_1^o}{\delta t}\right)_{ei}, \tag{34b}$$

with,

$$\left(\frac{\delta f_1^o}{\delta t}\right)_{en} = -\nu_{en}\, f_1^o,$$

$$\left(\frac{f_1^o}{t}\right)_{ei} = -\nu_{ei}\, f_1^o.$$

$$B_1^1 = \left(\frac{\delta f_1^1}{\delta t}\right)_{en} + \left(\frac{\delta f_1^1}{\delta t}\right)_{ei}, \qquad (34c)$$

with,

$$\left(\frac{\delta f_1^1}{\delta t}\right)_{en} = -\nu_{en}\, f_1^1,$$

$$\left(\frac{\delta f_1^1}{\delta t}\right)_{ei} = -\nu_{ei}\, f_1^1.$$

where, T_g is the temperature of the gas,

k is the Boltzmann's constant,

M is the mass of the heavy collision particle,

ν_{en} is the electron-neutral collision frequency,

ν_{ei} is the electron-ion collision frequency.

Now equations (34a), (34b) and (34c) show that the collision terms for both electron-neutral and electron-ion collisions are of the same form with the appropriate electron collision frequencies[13]. This leads to the conclusion that the total electron collision frequency may be simply a sum of the individual collision frequencies.

$$\nu_g = \nu_{en} + \nu_{ei} \qquad (35)$$

Using this approach in connection with equations (34b) and (34c), equations (33b) and (33c) may be written:

$$v \tilde{\nabla}_r f_0^o - \tilde{\omega}_b \times \tilde{f}_1^o = - \nu_{en} \tilde{f}_1^o - \nu_{ei} \tilde{f}_1^o, \tag{36a}$$

$$= - \nu_g \tilde{f}_1^o .$$

$$j\omega \tilde{f}_1^1 + \tilde{a}^1 \frac{d}{dv} f_0^o - \tilde{\omega}_b \times \tilde{f}_1^1 = - \nu_{en} \tilde{f}_1^1 - \nu_{ei} \tilde{f}_1^1, \tag{36b}$$

$$= - \nu_g \tilde{f}_1^1.$$

The electron current density ($\tilde{J}$) can now be used with equations (36a) and (36b) to calculate the conductivity of the medium. Expansion of the current density yields

$$\tilde{J} = \tilde{J}^o + \tilde{J}^1 e^{j\omega t}, \tag{37}$$

where, $\tilde{J}^o$ is the dc current density,

$\tilde{J}^1$ is the ac current density.

The ac current density may be written

$$\tilde{J}^1 = \sum qN \langle \tilde{v} \rangle, \tag{38}$$

where, q is the charge of the particle,

$\langle \tilde{v} \rangle$ is the average particle velocity,

N, the number of particles, is the normalization factor for the electron distribution function.

$$N = 4\pi \int_0^\infty f \, v^2 \, dv \tag{38a}$$

For the ionosphere, q = e and

$$\langle \tilde{v} \rangle = \frac{4\pi}{3N} \int_0^\infty \tilde{f}_1^1 \, v^3 \, dv \quad , \tag{39}$$

therefore, $$\tilde{J}^1 = \frac{4\pi e}{3}\int_0^\infty \tilde{f}_1^1 \, v^3 \, dv \quad . \tag{40}$$

The a-c conductivity $\{\sigma\}$ is given by

$$\tilde{J}^1 = \{\sigma\} \; \tilde{E}^1 , \tag{41}$$

and arises from the a-c component of the electric field $\tilde{E}^1$.

Since the d-c conductivity is not a function of the received radio frequency (ω) and if the electric and magnetic field are assumed to be sufficiently uniform so that their gradients are small enough not to disturb the ionospheric equilibrium, then the a-c component of the conductivity is the only term in our expansions which must be considered in ionospheric absorption.

Consider equation (36b),

$$j\omega \tilde{f}_1^1 + \tilde{a}^1 \frac{d}{dv} f_0^0 - \tilde{\omega}_b \times \tilde{f}_1^1 = -\nu_g \tilde{f}_1^1 \quad .$$

This equation may be solved in matrix form for the a-c conductivity[12].

Rearranging,

$$\left[(\nu_g + j\omega) - \tilde{\omega}_b \times\right] \tilde{f}_1^1 = -\tilde{a}^1 \frac{df_0^0}{dv} \quad , \tag{42}$$

$$\begin{aligned}\left[(\nu_g + j\omega) + \tilde{\omega}_b \times\right]\left[(\nu_g + j\omega) - \tilde{\omega}_b \times\right] &= \\ -\left[(\nu_g + j\omega) + \tilde{\omega}_b \times\right] \tilde{a}^1 \frac{df_0^0}{dv} \quad ,\end{aligned} \tag{43}$$

$$(\nu_g + j\omega)^2 \tilde{f}_1^1 - \tilde{\omega}_b \times \tilde{\omega}_b \times \tilde{f}_1^1 = -\left[(\nu_g + j\omega) + \tilde{\omega}_b \times\right] \tilde{a}^1 \frac{df_0^\circ}{dV} \quad , \tag{44}$$

$$(\nu_g + j\omega)^2 \tilde{f}_1^1 - \left[(\tilde{\omega}_b \cdot \tilde{f}_1^1)\, \tilde{\omega}_b - (\tilde{\omega}_b \cdot \tilde{\omega}_b)\, \tilde{f}_1^1\right] =$$

$$-\left[(\nu_g + j\omega) + \tilde{\omega}_b \times\right] \tilde{a}^1 \frac{df_0^\circ}{dV} \quad . \tag{45}$$

Assuming the magnetic field to be in the z-direction,

$$\tilde{\omega}_b \rightarrow \omega_b\ \hat{k}.$$

The component vector equations are

$$\left[(\nu_g + j\omega)^2 + \omega_b^2\right] f_{1x}^1 =$$

$$-(\nu_g + j\omega)\, a_x^1 \frac{df_0^\circ}{dV} - \omega_b a_y^1 \frac{df_0^\circ}{dV} \quad , \tag{46a}$$

$$\left[(\nu_g + j\omega)^2 + \omega_b^2\right] f_{1y}^1 =$$

$$-(\nu_g + j\omega)\, a_y^1 \frac{df_0^\circ}{dV} + \omega_b a_x^1 \frac{df_0^\circ}{dV} \tag{46b}$$

$$\left[(\nu_g + j\omega)^2\right] f_{1z}^1 = -(\nu_g + j\omega)\, a_z^1 \frac{df_0^\circ}{dV} \tag{46c}$$

or, in matrix form,

$$\tilde{f}_1^1 = -\left\{M\right\} \cdot \tilde{a}^1 \frac{df_0^\circ}{dV} \tag{47}$$

where,

$$\left\{M\right\} = \frac{1}{\left[(\nu_g + j\omega)^2 + \omega_b^2\right]} \begin{pmatrix} \nu_g + j\omega & \omega_b & 0 \\ -\omega_b & \nu_g + j\omega & 0 \\ 0 & 0 & \dfrac{(\nu_g + j\omega)^2 + \omega_b^2}{\nu_g - j\omega} \end{pmatrix} . \tag{47a}$$

Employing equations (40) and (41) with equation (47),

$$\{\sigma\} = \frac{4\pi e}{3}\int_0^\infty \tilde{f}_1^1 \, v^3 dv \Big/ \tilde{E}^1 \quad , \tag{48}$$

and,

$$\{\sigma\} = \frac{4\pi e}{3}\int_0^\infty \left(-\{M\}\cdot \tilde{a}^1 \frac{df_0^0}{dv}\right) v^3 dv \Big/ \tilde{E}^1 \quad . \tag{49}$$

Referring to equation (33),

$$\tilde{a}^1 = \frac{e}{m}\tilde{E}^1,$$

so that,

$$\{\sigma\} = -\frac{4\pi e^2}{3m}\int_0^\infty \{M\} v^3 \frac{df_0^0}{dv}\, dv \quad . \tag{50}$$

The a-c conductivity matrix may be written now as

$$\{\sigma\} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix} \tag{51}$$

or, combining with equations (47a) and (50)

$$\sigma_{11} = \sigma_{22} = -\frac{4\pi e^2}{3m}\int_0^\infty \frac{\nu_g + j\omega}{(\nu_g + j\omega)^2 + \omega_b^2} v^3 \frac{df_0^0}{dv}\, dv \quad , \tag{52a}$$

$$-\sigma_{21} = \sigma_{12} = -\frac{4\pi e^2}{3m}\int_0^\infty \frac{\omega_b}{(\nu_g + j\omega)^2 + \omega_b^2} v^3 \frac{df_0^0}{dv}\, dv \quad , \tag{52b}$$

$$\sigma_{33} = -\frac{4\pi e^2}{3m}\int_0^\infty \frac{\nu_g + j\omega}{\nu_g^2 + \omega^2} v^3 \frac{df_0^0}{dv}\, dv \quad . \tag{52c}$$

IV. GENERALIZED FORMULA FOR IONOSPHERIC ABSORPTION

The formula for the a-c conductivity can be used now with equation (27) to determine the absorption due to the ionosphere.

$$D = 8.69 \int_0^\infty \frac{2\pi}{c} \{\sigma\}_{real} \, ds \quad . \tag{53}$$

Now, if the z-direction is taken vertical to the antenna site, then

$$\{\sigma\}_{real} = (\sigma_{33})_{real} \tag{54}$$

This can be justified as follows: Most observing sites can be either located at high geomagnetic latitudes or their receiving frequencies can be sufficiently high enough to make the gyromagnetic frequency negligibly small in comparison. In either case the matrix conductivity can be replaced by the z-component if the antenna receives along the vertical direction. Substituting equation (52c) into equation (53) using these assumptions results in the following:

$$D = -8.69 \int_0^\infty \frac{2\pi}{c} \left(\frac{4\pi e^2}{3m} \int_0^\infty \frac{\nu_g + j\omega}{\nu_g^2 + \omega^2} \, v^3 \frac{df_0^o}{dv} \, dv \right)_{real} ds \tag{54}$$

The electron distribution (f_0^o) has yet to be specified and is, so far, only restricted to a form in which the velocity dependence can be separated from any other dependencies[13].

If very few electrons are excited to high energies where inelastic collisions are dominant[14], then the distribution function may be written:

$$f_0^0 = A \exp\left[-\int_0^v \left\{kT_g + \frac{2e^2E^2}{3m\xi[\langle\nu_g\rangle^2+\omega^2]}\right\}^{-1} mv\,dv\right] \tag{55}$$

where, ξ is the fraction of the electron low-energy collision losses. If the electric field is small as in the normal ionosphere, equation (55) will reduce to a Maxwellian velocity distribution.

$$f_0^0 = A \exp\left\{-\frac{mv^2}{2kT_g}\right\} \tag{56}$$

where, A is the normalization constant,

T_g is the neutral gas temperature,

m is the electron mass,

k is the Boltzmann's constant.

In order to normalize the distribution function, the ionosphere must be assumed to be in an equilibrium state such that

$$T \approx T_g, \tag{57a}$$

$$N \approx N_+, \tag{57b}$$

where, T is the electron temperature,

N is the electron concentration,

N_+ is the ion concentration.

Under these assumptions, the distribution function can be normalized to the electron concentration.

$$f_0^0 = N\left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) \tag{58}$$

Therefore, equation (54)

$$\frac{df_0^\circ}{dV} = -\frac{2N}{\pi^{3/2}}\left(\frac{m}{2kT}\right)^{5/2} V \exp\left(-\frac{mV^2}{2kT}\right) \tag{59}$$

and,

$$D = 8.69\int_0^\infty \frac{2\pi}{c}\left[\frac{8Ne^2}{3m\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{5/2}\right.$$
$$\left.\int_0^\infty \frac{\nu_g + j\omega}{\nu_g^2 + \omega^2} V^4 \exp\left(-\frac{mV^2}{2kT}\right) dv\right]_{real} ds \tag{60}$$

or, rewriting equation (60)

$$D = 8.69\, A\int_0^\infty N T^{-5/2}\int_0^\infty \left(\frac{\nu_g + j\omega}{\nu_g^2 + \omega^2}\right)_{real} V^4 \exp\left(-\frac{mV^2}{2kT}\right) dv\, ds \tag{61}$$

where, $A = \frac{2\pi}{c}\left(\frac{8e^2}{3\sqrt{\pi}\,m}\right)\left(\frac{m}{2k}\right)^{5/2}$.

Taking only the real part of equation (61) and defining a new parameter α,

$$\alpha = \left(\frac{m}{2kT}\right), \tag{61a}$$

then,

$$D = 8.69\, A_0 \int_0^\infty N\alpha^{5/2}\int_0^\infty \frac{\nu_g}{\nu_g^2 + \omega^2} v^4 e^{-\alpha v^2}\, dvds, \tag{62}$$

where, $A_0 = \frac{16\, e^2\sqrt{\pi}}{3\, m\, c}$.

V. EFFECTIVE COLLISION FREQUENCY AND SCATTERING CROSS SECTION MEASUREMENTS

In order to calculate the ionospheric absorption, it is necessary to know the dependence of the collision frequency on velocity. The collision frequency ($\mathcal{V}_g$) is given by:[10]

$$\nu_g(v) = \sum_\beta \nu_{g\beta}(v) = \sum N_\beta v\, Q_{d_\beta}(v) \qquad (63)$$

where, β is the particle species interacting with the electrons,

N is the β particle concentration,

Q_{d_β} (V) is the scattering cross section of β particle.

In principle, the scattering cross sections can be measured directly or computed from theory. However, theoretical calculations are so complex that approximations must be made which severely limit their usefulness in the case of the ionosphere. Most cross sections can be derived experimentally from microwave conductivity, electron drift velocity, and electron diffusion measurements, provided that the electron collision frequency can be expressed as a monomial in a power of velocity[10].

Experiments have shown that the ionosphere is populated by many species of ions and molecules. However, for heights exceeding 50 Km, the partial collision frequency of most species does not contribute to the total collision frequency within the accuracy of the absorption calculation. The ionospheric absorption occurs mainly due to collisions between electrons and neutral nitrogen and oxygen molecules, and collisions due to electrons and molecular or atomic ions.

However, the role of atomic oxygen in ionospheric absorption is quite uncertain. Measurements of the electron scattering cross section of atomic oxygen are extremely difficult[10], and only a probable upper and lower limit has been found[15][16]. The density of atomic oxygen in the ionosphere is also uncertain[17]. Recent investigations have preferred the lower scattering cross section value as well as the lower limit on concentration [10] [18]. If this proves correct, atomic oxygen will make a negligible contribution to ionospheric absorption.

In the ionospheric absorption calculation here, only those electron collisions involving ions, and neutral nitrogen and oxygen molecules will be considered.

Cross sections for slow electrons colliding with neutral nitrogen molecules have been determined by several experimenters[19][20][21].

$$Q_{dN_2}(v) = \alpha_{N_2} v \qquad (cm^2), \tag{64}$$

where, $\alpha_{N_2} = 2.34 \times 10^{-23}$.

It is unlikely that this value is in error by more than 10%[10].

Similarily, cross sections for collisions with neutral molecular oxygen have been found[22][23].

$$Q_{dO_2}(v) = \alpha_{O_2} v \qquad (cm^2), \tag{65}$$

where, $\alpha_{O_2} = 2.5 \times 10^{-23}$.

Errors as high as 100% may be expected here[10].

The differential scattering cross section for collisions with ions can be found from the Rutherford scattering formula, but because of the infinite range of the Coulomb potential, the electron-ion scattering cross sections are infinite. However, in a slightly ionized or neutral plasma[10] the coulomb interaction of the electron with distant ions is cut off due to shielding by other free electrons. This problem then can be treated in a Boltzmann analysis[24][13].

$$Q_{di} = \alpha_{ei}/v^4 \qquad (cm^2), \tag{66}$$

where,

$$\alpha_{ei} = 4\pi\left(\frac{Ze^2}{4\pi\epsilon_0 m}\right)^2 \ln\Lambda ,$$

and, $$\Lambda = \frac{3}{2Ze^3}\,\frac{(4\pi\epsilon_0 kT)^{3/2}}{(\pi N)^{1/2}} ,$$

with Z the charge numbers of the ions,

ϵ_0 the dielectric constant of free space.

If the ions are taken as singly charged, as is approximately true in the ionosphere, equation (66) may be evaluated in cgs units.

$$\alpha_{ei} = 8.00 \times 10^{17} \ln\Lambda. \tag{66a}$$

$$\Lambda = 1.24 \times 10^4\, T^{3/2}/N^{\frac{1}{2}}. \tag{66b}$$

Employing equation (63) with equations (64), (65), (66),

$$\nu_g(v) = \alpha_{N_2} v^2 N_2 + \alpha_{O_2} v^2 O_2 + \alpha_{ei} N_+ v^{-3}, \tag{67}$$

where, N_2 is the nitrogen molecule concentration,

O_2 is the oxygen molecule concentration,

N_+ is the ion concentration.

VI. A MODEL OF NORMAL IONOSPHERIC ABSORPTION

Equation (62) gives the ionospheric absorption in terms of a collision frequency integrated over a velocity space. If equation (67) represents the velocity dependence of the collision frequency, then the integration can in principle be performed.

$$D = 8.69 A_0 \int_0^\infty N \alpha^{5/2} \int_0^\infty \frac{\alpha_{N_2} v^2 N_2 + \alpha_{O_2} v^2 O_2 + \alpha_{ei} v^{-3} N_+}{\left(\alpha_{N_2} v^2 N_2 + \alpha_{O_2} v^2 O_2 + \alpha_{ei} v^{-3} N_+\right)^2 + \omega^2} v^4 e^{-\alpha v^2} \, dv \, ds \tag{68}$$

Equation (68) is ungainly, and the integration difficult; however, it can be shown that the integral may be broken up into parts and simplified under the conditions imposed by the ionosphere.

First, a model of the normal ionosphere in both composition and temperature must be constructed to evaluate equation (68). In figure (1), the temperature is shown as a function of altitude. The data is derived from the COSPAR 1961 International reference atmosphere[25] with some data points inferred from various standard meteorological measurements. Figure (2) gives the concentration of molecular nitrogen and oxygen as a function of altitude as derived from recent rocket and satellite records[17][10]. The electron density profile for the ionosphere as inferred from various rocket and satellite measurements is shown in figure (3)[17][10][26].

A most desirable situation for simplifying equation (68) would be a separation of electron-neutral collisions from electron-ion interactions. In order to evaluate such an approach, the average collision frequency for the two processes is calculated. Employing the real part of equation (52c), a high frequency, effective collision frequency $\langle\nu_g\rangle$, which is equivalent to that given in the Appleton-Hartree treatment [6] as independent of velocity, may be defined.

$$\left(\frac{m\,\sigma_{33}}{Ne^2}\right) = -\frac{4\pi}{3N}\int_0^\infty \frac{\nu_g(v)}{\nu_g^2(v)+\omega^2}\, v^3 \frac{df_0^\circ}{dv}\, dv \tag{69a}$$

$$\left(\frac{m\sigma}{Ne^2}\right)_{AH} = \frac{\langle\nu_g\rangle}{\langle\nu_g\rangle^2+\omega^2} \tag{69b}$$

Expanding, with $\omega \gg \nu_g$,

$$-\frac{4\pi}{3N}\int_0^\infty \frac{\nu_g(v)}{\omega^2}\, v^3 \frac{df_0^\circ}{dv}\, dv = \frac{\langle\nu_g\rangle}{\omega^2}, \tag{70}$$

or, the effective collision frequency,

$$\langle\nu_g\rangle = -\frac{4\pi}{3N}\int_0^\infty v^3\, \nu_g(v)\, \frac{df_0^\circ}{dv}\, dv \tag{71}$$

Substituting equations (59) and (61a),

$$\langle \nu_g \rangle = \frac{8}{3\sqrt{\pi}} \alpha^{5/2} \int_0^\infty v^4 \nu_g(v)\, e^{-\alpha v^2} dv \, . \tag{72}$$

For electron-neutral collisions, the effective collision frequency is found using the first part of equation (67),

$$\nu_{en} = \alpha_m v^2 \tag{73}$$

where,

$$\alpha_m = \alpha_{N_2} N_2 + \alpha_{O_2} O_2 \, . \qquad (64), (65)$$

Then,

$$\langle \nu_{en} \rangle = \frac{8}{3\sqrt{\pi}} \alpha^{5/2} \alpha_m \int_0^\infty v^6 e^{-\alpha v^2} dv \, , \tag{74}$$

and,

$$\langle \nu_{en} \rangle = \frac{5}{2} \frac{\alpha_m}{\alpha} \, , \tag{75}$$

or, in cgs units,

$$\langle \nu_{en} \rangle = 7.58 \times 10^{11} \alpha_m T \tag{75a}$$

For electron-ion interaction, equation (67) gives

$$\nu_{ei} = \alpha_i v^{-3} \tag{76}$$

where,

$$\alpha_i = \alpha_{ei} N_+ \, . \tag{66a}$$

Then,

$$\langle \nu_{ei} \rangle = \frac{8}{3\sqrt{\pi}} \alpha^{5/2} \alpha_i \int_0^\infty v\, e^{-\alpha v^2} dv \tag{77}$$

and,

$$\langle \nu_{ei} \rangle = \frac{4}{3\sqrt{\pi}} \alpha^{3/2} \alpha_i \tag{78}$$

or, in cgs units,

$$\langle \nu_{ei} \rangle = 4.44 \times 10^{-18} \alpha_i T^{-3/2} \, . \tag{78a}$$

The results of such a calculation are presented in figure (4) for the assumed model ionosphere. The graph shows that the proposed division may indeed be correct as the intersection of the two curves lies in a 'valley' below the maxima of the functions.

Assuming that this division exists, equation (68) may be separated into two parts depending on the type of collision.

I. Absorption due to electron-neutral collisions:

$$D_{en} = 8.69\, A_o \int_0^\infty N \alpha^{5/2} \int_0^\infty \frac{\alpha_m v^6 e^{-\alpha v^2}}{\alpha_m^2 v^4 + \omega^2}\, dv\, ds \tag{79}$$

II. Absorption due to electron-ion collisions:

$$D_{ei} = 8.69\, A_o \int_0^\infty N \alpha^{5/2} \int_0^\infty \frac{\alpha_i v e^{-\alpha v^2}}{\alpha_i^2 v^{-6} + \omega^2}\, dv\, ds \tag{80}$$

The integrals of equations (79) and (80) are easily performed when the Dingle functions are applied[27].

$$\mathfrak{C}_p(y) = (p!)^{-1} \int_0^\infty \frac{\epsilon^p e^{-\epsilon}}{\epsilon^2 + y^2}\, d\epsilon \tag{81a}$$

$$\mathfrak{F}_p(y) = (p!)^{-1} \int_0^\infty \frac{\epsilon^p e^{-\epsilon}}{1 + y \epsilon^3}\, d\epsilon \tag{81b}$$

If in equation (79),

$$\epsilon = \alpha v^2 ,$$

$$y = \frac{\alpha \omega}{\alpha_m} ,$$

then,

$$D_{en} = 8.69\, A_o \int_0^\infty \frac{N \alpha^{3/2}}{2 \alpha_m} \int_0^\infty \frac{\epsilon^{5/2} e^{-\epsilon}}{\epsilon^2 + y^2}\, d\epsilon\, ds , \tag{82}$$

or, in terms of the Dingle Integral (81a),

$$D_{en} = 8.69\, A_0 \int_0^\infty \frac{N\alpha^{3/2}}{2\alpha_m} \left(\frac{5}{2}!\right) \mathfrak{F}_{2.5}(y)\, ds \quad . \tag{83}$$

Evaluating the constants in cgs units yields

$$D_{en} = 8.69\, A_1 \int_0^\infty \frac{N}{\alpha_m T} \mathfrak{F}_{2.5}(y)\, ds \quad , \tag{84}$$

where,

$$A_1 = 4.12 \times 10^{-13} \quad ,$$

$$y = \frac{3.30 \times 10^{-12}\, \omega}{\alpha_m T} \quad .$$

Similiarly, if in equation (80),

$$\epsilon = \alpha V^2 \quad ,$$

$$y = \frac{\omega^2}{\alpha_i^2 \alpha^3} \quad ,$$

then,

$$D_{ei} = 8.69\, A_0 \int_0^\infty \frac{N}{2\alpha_i \alpha^{3/2}} \int_0^\infty \frac{\epsilon^3 e^{-\epsilon}}{1 + y\epsilon^3}\, d\epsilon\, ds \quad , \tag{85}$$

or, in terms of the Dingle Integral (81b),

$$D_{ei} = 8.69\, A_0 \int_0^\infty \frac{N}{2\alpha_i \alpha^{3/2}} (3!)\, \mathfrak{F}_3(y)\, ds \quad . \tag{86}$$

Evaluating the constants in cgs units yields

$$D_{ei} = 8.69\, A_2 \int_0^\infty \frac{N T^{3/2}}{\alpha_i} \mathfrak{F}_3(y)\, ds \quad , \tag{87}$$

where,

$$A_2 = 3.91 \times 10^{16} \quad ,$$

$$y = \frac{2.79 \times 10^{34}\, \omega^2 T^3}{\alpha_i^2} \quad .$$

Using equations (84) and (87), the ionospheric absorption can be calculated as a function of altitude. Figure (5) shows a calculated absorption curve using a frequency (f) of 24.3 Mc/sec and representing the integral of the path length as a step function 10 kilometers wide (ds). (For absorption at 10-120 Mc/sec, see Figure 7.).

The assumed division is again appropriate as the intersection of the absorption curves lies in a deep 'valley' well below the maximum absorption level. An addition of the two absorption curves (figure 5) results in no measurable increase in the total absorption and very little increase in the level of the 'valley' absorption.

From this data one must conclude that the ionospheric absorption results from two distinct processes: electron-neutral collisions which are confined to the lower ionosphere and electron-ion collisions which operate in the upper ionosphere.

VII. COSMIC RADIO NOISE METHOD FOR EXPERIMENTAL ANALYSIS OF IONOSPHERIC ABSORPTION

One is now in a position to critically evaluate the cosmic radio noise method of measuring the ionospheric absorption[5].

The ionospheric absorption of extraterrestrial radio waves as measured from an earth station is the sum of the contribution of all regions of the ionosphere: D region, E region, sporadic E region, F_1 region, F_2 region and possibly a region above F_2. Mitra and Shain[5], in their original work, found that the major contribution to the ionosphere absorption takes place in the D and F_2 regions. The absorption curve in figure (5) confirms that this is indeed very probable, and furthermore, that the division of equation (68) into two separate integral forms corresponds to a separation of the absorption into the D region which is controlled by electron-neutral collisions and the F_2 region controlled by electron-ion collisions. Such a division has been confirmed by other investigators[28].

The ionospheric absorption as defined in equation (1) is found from the comparison of the cosmic radio power (P) actually received by a fixed receiving system with the 'standard' power (P_o) received on the same system at the same sidereal time under conditions of negligible ionospheric absorption. Variations in P_o will result from changes in antenna direction and sidereal time due to the passage over the antenna site of the varying extraterrestrial radio noise[1]. If values of P_o can be determined for the entire sidereal day for each antenna beam direction, then the variations of P_o can be eliminated. Mitra and Shain state that if continuous observations are made for a period of one year, it will be possible to select observations for which ionospheric absorption is negligible for all sidereal times, that is when the electron content is low (the minimum useable frequency is low).

In figure (6) the absorption has been plotted as a function of altitude for a normal night ionosphere. The total absorption is much less than the day absorption of figure (5). Since most present-day receiving equipment used in absorption experiments can measure to an accuracy of 0.01 decibels, the night absorption of 0.005 db. can not be measured, whereas the day absorption of 0.779 db. is readily measurable. Clearly then, a 'standard' curve of P_0 as a function of sidereal time is a possible as well as an accurate comparison of P and P_0 for moderate absorption.

VIII. SEPARATION OF ABSORPTION REGIONS

It would be highly desirable to be able to separate the contributions to the absorption of the F_2 and D regions. Equation (87) in terms of the expected values of the ionospheric parameters reduces to an interesting relation.

$$D_{ei} = 8.69\, A_2 \int_0^\infty \frac{N T^{3/2}}{\alpha_i}\, \mathfrak{G}_3(y)\, ds \tag{87}$$

The Dingle integral may be expanded for large values of the parameter (y)[29].

$$\mathfrak{G}_3(y) = \frac{1}{y}\left[0.167 - 0.201\left(\frac{1}{y^{1/3}} - \frac{1}{y^{2/3}}\right) + \frac{0.004}{y} - \frac{0.018}{y}\log\frac{1}{y}\right]. \tag{88}$$

For the ionosphere and for receiving frequencies of the order of 10-120 Mc/sec,

$$\mathfrak{S}_3(y) = 0.167/y \qquad (89)$$

where,

$$y = \frac{2.79 \times 10^{34} \omega^2 T^3}{\alpha_i^2} ,$$

$$\alpha_i = 8.00 \times 10^{17} N_+ \ln 1.24 \times 10^4 T^{3/2}/N^{1/2} .$$

The natural logarithm in the expression for α_i does not vary greatly for ionospheric calculations so that α_i is approximately proportional only to the ion concentration N_+. Therefore,

$$D_{ei} \propto \frac{1}{\omega^2}\int_0^\infty \frac{N N_+}{T^{3/2}} \, ds \qquad . \qquad (90)$$

In the F_2 region where the electron-ion collisions dominate, the region is approximately isothermal, and under equilibrium conditions the electron and ion concentrations are equal so that the absorption in this region is essentially an intergration over the square of the electron content of the F_2 region (N^2).

Such an expansion of the Dingle intergal of equation (84) can not be made without introducing sizable errors at altitudes below 80 km.

$$D_{en} = 8.69 A_1 \int_0^\infty \frac{N}{\alpha_m T} \mathfrak{S}_{2.5}(y) \, ds \qquad (84)$$

Above 80 km[29],

$$\mathfrak{S}_{2.5}(y) = \frac{1}{y^2}\left[1 - 15.75/y^2\right] , \qquad (91)$$

or, for the ionosphere,

$$\mathfrak{S}_{2.5}(y) = 1/y^2 \quad , \tag{92}$$

where,

$$y = \frac{3.30 \times 10^{-12}\,\omega}{\alpha_m T} \,,$$

$$\alpha_m = 2.34 \times 10^{-23} N_2 + 2.5 \times 10^{-23} O_2 \quad .$$

Therefore,

$$D_{en} \propto \frac{1}{\omega^2}\int_0^{\infty} N(N_2 + O_2)\,T \; ds \quad . \tag{93}$$

Mitra and Shain[5] suggest that a correlation exists between F_2 region absorption and the critical frequency of the F_2 region. Many investigators use this property to separate the contributions to absorption from the different regions. However, equations (90) and (93) oppose the accuracy of this method. Since the maximum electron concentration is proportional to the critical frequency, a correlation between absorption and critical frequency is to be expected on the basis of equations (90) and (93); however, a separation of the F_2 region employing the critical frequency must include some D region absorption. However, it must be pointed out that since the F_2 region absorption is more sensitive to the electron concentration (N^2), then a separation based on the critical frequency will result in a good qualitative description of the absorption in each region.

Mitra and Shain[5] report also no dependence of absorption on the various ionospheric height measurements. The model given here supports these observations. The absorption curves (figures 5 and 6) do not differ in profile, but the difference in absorption between day and night is a uniform reduction in absorption at every height plus a loss of most of the D region absorption at night.

IX. CONCLUSION

The ionospheric absorption of extraterrestrial radio waves has been shown to be an integration of the real part of the complex ionospheric conductivity over the ray path. The conductivity tensor in an ionized medium has been derived in terms of an electron collision frequency which is dependent on the velocity of the particles.

Calculations employing a model of the normal, undisturbed ionosphere have shown that under equilibrium conditions only electron interactions with ions and collisions with neutral nitrogen and oxygen molecules contribute significantly to ionospheric absorption. Separate evaluations of the absorption due to electron-neutral collisions, and that due to electron-ion interactions are presented. Such calculations have shown that electron-neutral collisions dominate D region absorption and electron-ion interactions are predominate in F region absorption.

The total absorption measured by a ground-based antenna is found to be just the sum of the D and F region absorption. Calculations of absorption for both a normal day and night ionosphere have shown that absorption can be measured by comparing the actual power received with the power received at the same sidereal time under conditions of negligible absorption.

Present methods of separating the D and F regions from riometer data are found to be quantitatively unsound, although qualitative data can be obtained. Further work is needed in order to find a quantitative method of region separation. It may be possible to employ equation (90) with satellite measurements and theoretical electron density distribution functions[30] to separate the absorption due to the F region from that due to the D region.

ACKNOWLEDGEMENT

The author wishes to express his sincere thanks to Dr. A. C. Aikin for suggesting the problem and giving many useful suggestions. Thanks are also due to Dr. R. G. Stone, Dr. S. Chandra, and Mr. J. K. Alexander, Jr. for many helpful discussions.

APPENDIX A

The Measurement of Ionospheric Absorption Using Extraterrestrial Radio Waves

The cosmic radio noise[5] method of measuring ionospheric absorption compares the power of the extra-terrestrial radio waves actually received by a fixed receiving system with the power received on the same system at the same sidereal time under conditions of negligible ionospheric absorption.

$$D = -10 \log \frac{P}{P_o}, \qquad (1)$$

where, D is the ionospheric absorption in decibels,

P is the actual received power,

P_o is the received power with negligible ionospheric absorption.

The reception of random noise signals from diffused sources is often conveniently expressed by the equivalent antenna temperature[7]. A matched antenna in an enclosure at temperature (T^ok) will deliver a power

$$P = T\ k\ B \qquad (2)$$

in an observing bandwidth (B), where k is Boltzman's constant. Similarly, an equivalent radio noise temperature (T_e) can be defined

$$P_n = T_e \; k \; B, \tag{3}$$

where, P_n is the power of the extraterrestrial radio waves.

Consider a receiving system whose antenna beam looks at a region of the heavens whose radio temperature in the absence of ionospheric effects is (T_1). The antenna signal power will be

$$P_1 = T_1 \; k \; B. \tag{4}$$

Introducing the ionosphere into the full width of the antenna beam and assuming a uniform power transmission coefficient (α_2), the received power is reduced.

$$P_1 = \alpha_2 \; T_1 \; k \; B. \tag{5}$$

The ionosphere will radiate radio noise due to its temperature (T_2) in an amount determined by its absorbing ability.

$$P_2 = (1 - \alpha_2) \; T_2 \; k \; B. \tag{6}$$

The antenna signal is connected to the receiver with a matched transmission line at temperature (T_3) and transmission coefficient (α_3). This transmission line will act both as an attenuator (7a) and noise source (7b). (Equations (4), (5) and (6)).

$$P = k \; B \alpha_3 \; (\alpha_2 \; T_1 + (1 - \alpha_2)T_2). \tag{7a}$$

$$P_3 = (1 - \alpha_3)T_3 \; k \; B. \tag{7b}$$

The received noise power will add to the noise power (P_4) of the receiver.

$$P_4 = (F - 1)\ T_A\ k\ B, \tag{8}$$

where, F is the receiver noise figure,

T_A is the ambient temperature.

The presence of interferring signals will add power (I) to the received noise also.

Assuming the receiver to have a power gain (G), the power output (P_R) may be written then as

$$P_R = G(P_1 + P_2 + P_3 + P_4 + I). \tag{9}$$

Since ionospheric absorption measurements are based on a comparison of the receiver output power on different days, the stability of the receiver gain critically affects the accuracy of the measurements.

A radiometer principle developed by Mackin, Ryle, and Vanberg[31] for radio astronomical work can be used to eliminate the gain instability. The Relative Ionospheric Opacity Meter or 'Riometer' is a self-balancing system in which the antenna power is electronically compared to the power of a noise diode. A servo-mechanism continuously varies the noise diode power so that the antenna and diode powers are equivalent. The noise diode is operated in a temperature limited state (i.e. all the emission current from the cathode reaches the plate) so that its noise power (P_{nd}) is proportional to the current flowing through it[32]:

$$P_{nd} = \overline{I}_N^2 \, R_o, \tag{10a}$$

and,

$$\overline{I}_N^2 = 2 \, e \, I \, B, \tag{10b}$$

where, $\overline{I}_N^2$ is the noise output current,

I is the current through the noise diode,

e is the electronic charge,

R_o is the effective resistance of the circuit with impedance matching.

Therefore, in order to measure the antenna power, only the noise diode current need be measured.

In such a system, the receiver becomes a null detector, and thus the power measurements are not affected by receiver gain instability. Equation (9) becomes under these conditions,

$$P_{nd} = P_1 + P_2 + P_3 + \Delta P_4 + I. \tag{11}$$

where, ΔP_4 is the difference between the equivalent noise input power of the receiver when connected alternately between the antenna and noise diode. Therefore, the noise diode output is a function only of the received powers[25].

A. Radio noise from the ionosphere: (P_2)

The received power from ionospheric radio noise will be small compared to the power from extraterrestrial sources for frequencies above the minimum ionospheric penetration frequency. From equations (5) and (6)

$$\frac{P_2}{P_1} = \frac{(1 - \alpha_2)T_2}{\alpha_2 T_1}. \tag{12}$$

At 20 Mc/sec, T_1 can be expected to be of the order of 10^5 °K(33) while electron temperatures in the ionosphere will not exceed 2,000 °K(25). The transmission coefficient is approximately, $\alpha_2 = 0.9$.

$$\frac{P_2}{P_1} = 2.2 \times 10^{-3}. \tag{13}$$

Therefore, P_2 can be neglected in comparison with P_1.

B. Radio noise from the transmission line: (P_3)

The received power will not require correction due to power generation or attenuation of the transmission line if the following precautions are taken (Equation (7b)).

(1) Since $P_3 \propto \alpha_3$, the transmission line should be constructed of low-loss cable and shielded from outside interference.

(2) Since $P_3 \propto T_3$, the transmission line should be kept at a fairly constant temperature.

It may be desirable to bury the main part of the transmission line and shield the remainder from direct sunlight.

C. Radio noise from interference: (I)

Since the extraterrestrial radio waves are quite weak, measurements are quite susceptible to interference (both man-made and natural). The instrument should be placed at a relativity interference-free site, and should be operated at frequencies where little interference is expected. The use of minimum signal detector circuits increases the probability of excluding interference.

D. Radio noise from switching circuits: (ΔP_4)

If the antenna impedance can be accurately reproduced by the noise diode, the errors due to switching are extremely small, and can easily be neglected.

If care is taken selecting the antenna site and receiving frequency, the noise diode current will essentially measure the extraterrestrial radio power with ionospheric absorption.

$$P_{nd} = P_1. \tag{14}$$

Equation (7a) with the results of equation (13) becomes

$$P_1 = \alpha_3 \alpha_2 T_1 k B. \tag{15}$$

Therefore, $$P_{nd} = \alpha_3 \alpha_2 T_1 k B. \tag{16}$$

If the transmission line coefficient (α_3) is constant, the ratio of two noise diode powers at the same sidereal time is

$$\frac{(P_{nd})_a}{(P_{nd})_b} = \frac{(\alpha_2)_a}{(\alpha_2)_b}. \tag{17}$$

where, $T_a = T_b = T_1$ since the same sidereal time requires the antenna look at the same region of the heavens with effective temperature T_1.

Employing equation (10),

$$\frac{(P_{nd})_a}{(P_{nd})_b} = \frac{I_a}{I_b}. \tag{18}$$

Therefore,

$$\frac{I_a}{I_b} = \frac{(\alpha_2)_a}{(\alpha_2)_b}. \tag{19}$$

From equation (19), one sees that any variation of the received extraterrestrial radio power (i.e. noise diode current) can be directly attributed to ionospheric absorption.

There are a few exceptions to this statement.

A. The received radio power from the sun when undisturbed is less than 1% of the cosmic noise, and can be neglected. However, when active sun spots are present, high signal powers will be detected if the sun is in the antenna beam.

B. The discrete sources Venus and Jupiter present varying powers when in the antenna beam, but for a wide-beam antenna these sources are undetectable.

C. Scintillations of signals from intense sources like Cygnus A can cause errors of one or two percent; however, integration by the receiving system removes these errors.

The ionospheric absorption as given by equation (1) is essentially a measurement of two noise diode currents.

$$D = -10 \log \frac{I}{I_o}, \qquad (20)$$

where, I is the actual measured diode current,

I_o is the measured diode current at the same sidereal time with the same instrument under conditions of negligible ionospheric absorption.

Figure (8) shows a typical riometer record. The record consists of the measured noise diode current as a function of time, a calibration spectrum every 24 hours, and time marks every hour. An evaluation of the record consists of a current reduction and a time reduction.

APPENDIX B

Reduction of Riometer Current Measurements

Commercially, riometers are constructed so that only very small drifts in calibration occur. The calibration is accomplished by substituting another noise diode for the antenna system. The calibration will be extremely accurate providing the test noise diode impedance matches that of the antenna system.

The calibration is automatically performed every 24 hours in five (5) test diode current steps and one (1) zero step. The steps are read from the record in a convenient system of chart units (i_o, i_1, i_2, i_3, i_4, i_5).

For example; (Figure 8) 2400 E.S.T. Aug. 1, 1963.

i_o = 0.60 c.u.

i_1 = 4.90 c.u.

i_2 = 6.75 c.u.

i_3 = 8.15 c.u.

i_4 = 9.20 c.u.

i_5 = 9.85 c.u.

These readings are reduced to values suitable for determining the power difference between calibration points by subtracting the zero point (i_o) from each of the other readings

$$R_1 = i_1 - i_o$$

$$R_2 = i_2 - i_o$$

$$R_3 = i_3 - i_o$$

$$R_4 = i_4 - i_o$$

$$R_5 = i_5 - i_o \qquad (21)$$

For example, from above

$$R_1 = 4.90 - 0.60 = 4.30 \text{ c.u.}$$

$$R_2 = 6.75 - 0.60 = 6.15 \text{ c.u.}$$

$$R_3 = 8.15 - 0.60 = 7.55 \text{ c.u.}$$

$$R_4 = 9.20 - 0.60 = 8.60 \text{ c.u.}$$

$$R_5 = 9.85 - 0.60 = 9.25 \text{ c.u.} \tag{21a}$$

These values correspond to the currents (I) flowing in the test diode for each calibration point.

For example,

$$I_1 = 1 \text{ ma.}$$

$$I_2 = 2 \text{ ma.}$$

$$I_3 = 3 \text{ ma.}$$

$$I_4 = 4 \text{ ma.}$$

$$I_5 = 5 \text{ ma.} \tag{21b}$$

A plot of the logarithm of the noise diode current as a function of the number of chart units is thus the calibration curve. The curve so determined is a straight line since the input power (decibels) is linear with noise diode current.

The straight line determined by the five calibration points can be found by an application of the least squares method.

Assume a straight line,

$$\ln I = p_1 + p_2 R, \tag{22}$$

or,

$$y_k = p_1 + p_2 R_k, \tag{23}$$

where, p_1 and p_2 are constants and

k is a particular data point.

The least squares starting conditions are (p_1^o, p_2^o, y_1^o).

$$p_1 = p_1^o + \Delta p_1. \tag{24a}$$

$$p_2 = p_2^o + \Delta p_2. \tag{24b}$$

Then, applying Taylor's series expansion,

$$y_{k,\ \text{curve}} = y_k^o + \left(\frac{\partial y_k}{\partial p_1}\right)_o \Delta p_1 + \left(\frac{\partial y_k}{\partial p_2}\right)_o \Delta p_2. \tag{24c}$$

For a straight line, this expansion terminates after the first order terms.

Define:

$$z = \sum_{k=1}^{N} (y_{k,\ \text{curve}} - y_{k,\ \text{obs}})^2, \tag{25}$$

where, N is the number of data points,

$y_{k,\ \text{obs}}$ is an observed value of ln I.

From equation (24c),

$$z = \sum_{k=1}^{N} \Big[(y_k^o - y_{k,\ \text{obs}})^2 + 2(y_k^o - y_{k,\ \text{obs}}) \left\{ \left(\frac{\partial y_k}{\partial p_1}\right)_o \Delta p_1 + \left(\frac{\partial y_k}{\partial p_2}\right)_o \Delta p_2 \right\} + \left(\frac{\partial y_k}{\partial p_1}\right)_o^2 (\Delta p_1)^2 + 2\left(\frac{\partial y_k}{p_1}\right)_o \left(\frac{\partial y_k}{\partial p_1}\right)_o \Delta p_1 \Delta p_2 + \left(\frac{\partial y_k}{\partial p_2}\right)_o^2 (\Delta p_2)^2 \Big]. \tag{26}$$

Minimizing the scatter of the data points from the accurate curve corresponds to minimizing the quantity Z with respect to Δp_1 and Δp_2.

$$\frac{\partial Z}{\partial(\Delta p_1)} = \sum_{k=1}^{N} \Big[2(y_k^o - y_{k,\ obs}) \left(\frac{\partial y_k}{\partial p_1}\right)_o$$

$$+ 2\left(\frac{\partial y_k}{\partial p_1}\right)_o^2 \Delta p_1 + 2\left(\frac{\partial y_k}{\partial p_1}\right)_o \left(\frac{\partial y_k}{\partial p_2}\right)_o \Delta p_2 \Big]$$

$$= 0. \qquad (27a)$$

$$\frac{\partial Z}{\partial(\Delta p_2)} = \sum_{k=1}^{N} 2(y_k^o - y_{k,\ obs}) \left(\frac{\partial y_k}{\partial p_2}\right)_o$$

$$+ 2\left(\frac{\partial y_k}{\partial p_1}\right)_o \left(\frac{\partial y_2}{\partial p_2}\right)_o \Delta p_1$$

$$+ 2\left(\frac{\partial y_k}{\partial p_2}\right)_o^2 \Delta p_2 = 0. \qquad (27b)$$

For starting conditions, let

$$p_1^o = p_2^o = 0,$$

so that $\quad y_k^o = 0.$

The partial derivatives may be evaluated,

$$\left(\frac{\partial y_k}{\partial p_1}\right) = 1,$$

$$\left(\frac{\partial y_k}{\partial p_2}\right) = R_k.$$

Then, equation (27) reduces to

$$\sum_{k=1}^{N} y_{k,\ obs} = \left\{ \sum_{k=1}^{N} (1)^2 \right\} \Delta p_1 + \left\{ \sum_{k=1}^{N} R_k \right\} \Delta p_2 , \tag{28a}$$

and,

$$\sum_{k=1}^{N} y_{k,\ obs}\ R_k = \left\{ \sum_{k=1}^{N} R_k \right\} \Delta p_1 + \left\{ \sum_{k=1}^{N} R_k^2 \right\} \Delta p_2 . \tag{28b}$$

Equation (28) may be solved by determinates.

$$p_1 = \Delta p_1 = \frac{1}{D} \begin{vmatrix} \sum_{k=1}^{N} y_{k,\ obs} & \sum_{k=1}^{N} R_k \\ \sum_{k=1}^{N} y_{k,\ obs}\ R_k & \sum_{k=1}^{N} R_k^2 \end{vmatrix} \tag{29a}$$

$$p_2 = \Delta p_2 = \frac{1}{D} \begin{vmatrix} N & \sum_{k=1}^{N} y_{k,\ obs} \\ \sum_{k=1}^{N} R_k & \sum_{k=1}^{N} y_{k,\ obs}\ R_k \end{vmatrix} \tag{29b}$$

where,

$$D = \begin{vmatrix} N & \sum_{k=1}^{N} R_k \\ \sum_{k=1}^{N} R_k & \sum_{k=1}^{N} R_k^2 \end{vmatrix}$$

and,

$$\sum_{k=1}^{N} (1)^2 = N.$$

For example, employing equations (29) to solve for equation (22) and using the values in examples (21a) and (21b):

$$\mathcal{Y}_k = \ln I = p_1 + p_2 R. \tag{22}$$

Solving equation (29),

$$D = N \sum_{k=1}^{N} R_k^2 - (\sum_{k=1}^{N} R_k)(\sum_{k=1}^{N} R_k), \tag{29a}$$

$$p_1 = \frac{1}{D}\Big[(\sum_{k=1}^{N} \mathcal{Y}_{k,\ obs})(\sum_{k=1}^{N} R_k^2) - (\sum_{k=1}^{N} \mathcal{Y}_{k,\ obs} R_k)(\sum_{k=1}^{N} R_k)\Big], \tag{29b}$$

$$p_2 = \frac{1}{D}\Big[N \sum_{k=1}^{N} \mathcal{Y}_{k,\ obs} R_k - (\sum_{k=1}^{N} \mathcal{Y}_{k,\ obs})(\sum_{k=1}^{N} R_k)\Big]. \tag{29c}$$

Using values from (21b) and (21a),

N	(R)	(R^2)	(I)	$(\ _{k,\ obs})$	$Y_{k,\ obs}\,R$
1	4.30 cu	18.5	1 ma	0	0
2	6.15 cu	37.8	2 ma	0.301	1.85
3	7.55 cu	57.0	3 ma	0.477	3.60
4	8.60 cu	74.0	4 ma	0.602	5.18
5	9.25 cu	85.6	5 ma	0.699	6.47
N=5	A=35.95	B=272.9	$\sum_{k=1}^{N} \ _{k,\ obs}$	=2.08	C=17.10

$$D=5(272.9) - (35.85)(35.85) = 79.3 \tag{29a}$$

$$p_1 = \frac{(2.08)(272.9) - (35.85)(17.10)}{79.3} = -0.573 \tag{29b}$$

$$p_2 = \frac{5(17.10) - (2.08)(35.85)}{79.3} = + 0.138. \tag{29c}$$

Equation (22) is therefore,

$$\ln I = -0.53 + 0.138 R.$$

The above equation can now be used to draw the calibration curve as shown in Figure (9).

$$R = \frac{\ln I - p_1}{p_2},$$

or

$$R = \frac{\ln I + 0.573}{0.138}$$

Using the points I = 6.0 and 0.6,

$$p_1(I,R) = p_1(6.0,\ 9.79),$$

$$p_2(I,R) = p_2(0.6,\ 2.54),$$

which determines the straight line.

Similarly, data from 2400 August 2, 1963 may be reduced and a calibration curve as in figure (10) may be obtained.

Using the above, two calibration curves which are 24 hours apart may be found as in figures (9) and (10). Assuming that the difference in the two calibration curves is small, a linear interpolation may be performed to reduce any chart reading to an accurate current reading.

In figure (8) if the chart reading at point R is (i), then the reduced chart reading as in equation (21) is

$$R = i - i_o', \tag{30}$$

but i_o' must be the interpolated value of the zero point between calibration chart A (figure 9) and calibration chart B (figure 10).

$$R = i - \left[(i_o)_A + \frac{t}{t_o} \left\{ (i_o)_B - (i_o)_A \right\} \right], \tag{31}$$

where, t is the time difference between the reading at R and the calibration chart A in hours,

t_o is the time difference between calibration chart A and chart B in hours (usually 24 hours),

$(i_o)_A$ is the zero point (i_o) on the calibration chart A at the earlier time,

$(i_o)_B$ is the zero point (i_o) on the calibration chart B at the later time.

For example,

0215 August 2, 1963,

$i = 8.45$ c.u.

From chart A, 2400 August 1, 1963,

$(i_o)_A = 0.60$ c.u.

From chart B, 2400 August 2, 1963,

$(i_o)_B = 0.65$ c.u.

$t = 2.25$ hours from 2400 August 1, 1963.

$t_o = 24$ hours from 2400 August 1, 1963.

$$R = 8.45 - \left[(0.60) + \frac{2.25}{24}\left\{(0.65) - (0.60)\right\}\right].$$

$R = 7.84$ c.u.

Using the reduced chart reading (R), the corresponding noise diode current can be found from chart A = $(I)_A$ and from chart B = $(I)_B$. The actual noise diode current is found from a linear interpolation between these two values.

$$I = (I)_A + \frac{t}{t_o}\left[(I)_B - (I)_A\right], \tag{32}$$

where, t and t_o have the same meanings as in equation (31).

For example, from above R = 7.84 c.u. from chart A

$(I)_A = 3.20$ ma.

From chart B $(I)_B = 3.10$ ma. Therefore,

$$I = (3.20) + \frac{2.25}{24}\left[(3.10) - (.20)\right],$$

$I = 3.19$ ma.

These current readings (I) correspond to the actual diode current values needed in equation (20).

APPENDIX C

Reduction of Riometer Time Measurements

The riometer records are recorded in Eastern Standard Time with time marks each hour. However, in order that the given times should be most useful in comparing with the position of the sun, passage of satellites and other measurements, conversions should be made into the appropriate units[34].

A. Standard Time:

The earth is divided into twenty-four time zones centered around certain longitudual meridians which are 15^{o} apart. The time throughout each zone is the same, i.e. it has been standardized. Eastern Standard Time (E.S.T.) has the Standard Meridian 75^{o} W. from the Greenwich (or 0^{o}) Meridian.

B. Local Mean Time (LMT):

The time measured by the passage of a mean sun over the Local Meridian (longitude) is the Local Mean Time. Such a mean sun is defined so that every day of the year is of exactly the same duration and is equivalent to the average solar day.

The Local Mean Time is obtained from the standard time by adding the longitude of the observing site east of the standard meridian.

For example,

The LMT of an observing site of longitude (λ) 75^{o} 28' 05" W., when the E.S.T. is $2^{h}\ 15^{m}\ 00^{s}$ A.M. ($02^{h}\ 15^{m}\ 00^{s}$), Aug. 2, 1963 can be found as follows:

Conversion of the longitude (λ) to a time measurement from an arc measurement using Table XII of the Ephemeris, 1963[35]. The Standard Meridian (λ_o) is 75^{o} for E.S.T.

$$\lambda = 75^{o}\ 28'\ 05'' = 5^{h}\ 01^{m}\ 52^{s}$$

$$\lambda_o = 75^{o}\ 00'\ 00'' = 5^{h}\ 00^{m}\ 00^{s}$$

Conversion to LMT:

Given EST Aug. 2, 1963	$2^{h}\ 15^{m}\ 00^{s}$
$-\ \lambda$ West of Greenwich	$5^{h}\ 01^{m}\ 52^{s}$ *
$+\ \lambda_o$ West of Greenwich	$5^{h}\ 00^{m}\ 00^{s}$ *
LMT Aug. 2, 1963	$2^{h}\ 13^{m}\ 08^{s}$

* This gives the longitude east of the Standard Meridian.

C. Local Apparent Time (LAT):

Local Apparent Time is the time measured by the true sun. Such time measurements are equivalent to determining the true place of the sun in the sky for each day of the year. The difference between the Local Apparent Time and the Local Mean Time is called the Equation of Time (E.T.) and can be found from an Ephemeris (Sun, 1963).

For example,

LMT Aug. 2, 1963	$2^h\ 13^m\ 08^s$
+ E.T. Aug. 2, 1963 (35)	$-0^h\ 06^m\ 16^s$
LAT Aug. 2, 1963	$2^h\ 06^m\ 52^s$

D. Universal Time (U.T.):

Universal Time is the Local Mean Time at the Greenwich (or 0^o) Meridian. Universal Time may be found by adding the longitude (λ) of the observing site West of Greenwich Meridian to the Local Mean Time.

For example,

LMT Aug. 2, 1963	$2^h\ 13^m\ 08^s$
+ λ West of Greenwich	$5^h\ 01^m\ 52^s$
U.T. Aug. 2, 1963	$7^h\ 15^m\ 00^s$

E. Sidereal Time (S.T.):

Sidereal Time is measured with respect to a fixed point in the heavens, the vernal equinox. The sidereal day is the time between two successive transits of the vernal equinox over the Local Meridian.

Considering this definition and the above definition of Local Mean Time, one observes that 24 mean hours will be $24^h\ 3^m\ 56^s.555$ sidereal time; therefore, each day a sidereal clock will gain $3^m\ 56^s.555$ S.T. in 24 mean hours. Using an Ephemeris the Local Sidereal Time may be found from the Universal Time.

For example,

U.T. = $7^h\ 15^m\ 00^s$ August 2, 1963

Greenwich S.T. at $0^h\ 00^m\ 00^s$ U.T. August 2, 1963 from U.T. - S.T. charts in the Ephemeris, 1963[35].	$20^h\ 39^m\ 28^s$
+U.T.	$+\ 7^h\ 15^m\ 00^s$
+In $7^h\ 15^m\ 00^s$ S.T. gains over U.T. (Table IX - Ephemeris 1963)	$+\ 0^h\ 01^m\ 11^s$
Required Greenwich S.T.	$3^h\ 55^m\ 39^s$
- West of Greenwich	$-\ 5^h\ 01^m\ 52^s$
Local Sidereal Time (LST)	$22^h\ 53^m\ 47^s$

The Local Sidereal Time can be referred to a standard Epoch. Such calculations show that the LST must be reduced by $50^s.6$ to reduce the data to Epoch 1950. If accuracy of the time readings is of the order of one minute, (LST-1^m) will give an accurate reduction to Epoch 1950 which will be accurate at least until 1968.

APPENDIX D

Reduction of Absorbtion Measurements

The riometer records are now reduced to actual current values (I) which can be used to find the absorption by equation (20). However, there must be some method of determining which current readings correspond to I_o, the measured diode current under conditons of negligible ionospheric absorption.

The ionospheric absorption can be shown to be dependent on the electron concentration. The electron concentration determines also the minimum radio frequency which will pass through the earth's ionosphere. Therefore, for the absorption to become negligible the electron concentration must be low; and thus, the minimum usable frequency must be low.

It can be shown that if the electron concentration at F_2 peak is of the order of 10^5 electrons/cm^3, the ionospheric absorption is less than 0.01 db; whereas if the concentration is 10^6 electrons/cm^3, the ionospheric absorption is higher by 10^2.

The minimum usable frequency is given by [36]:

$$N = 1.24 \times 10^{-8} f^2. \tag{33}$$

For, $N = 10^6$,

then, $f = 8.98$ Mc/sec.

For, $N = 10^5$,

then, $f = 2.84$ Mc/sec.

It is reasonable to assume that all diode current readings (I) taken when the minimum useable frequency is less than 3 Mc/sec will be unaffected by ionospheric absorption, and thus they can be used as values of I_o in equation (20). (Some investigators prefer less than 1.6 Mc/sec[37]).

CONCLUSION

The Riometer is a highly efficient instrument for measuring the ionospheric absorption. The absorption is readily obtainable from the records, and easily reduced for use with satellite and rocket measurements.

REFERENCES

1. Jansky, K. G., Nature, 132, 66, 1933.
2. Jansky, K. G., Proc. I.R.E. 25, 1517, 1937.
3. Little, C. G. and Leinbach, H., Proc. I.R.E. 47, 315, 1957.
4. Shain, C. A., Aust. J. Sci. Research A3, 258, 1951.
5. Mitra, A. P. and Shain, C. A., J. Atmos. Terrest. Phys., 4, #4-5, 204, 1953.
6. Appleton, E. V., J. IEE, 71, 642, 1932.
7. Little, C. G. and Leinbach, H., Proc. I.R.E. 46, 334, 1958.
8. Jenkins, F. A. and White, H. E., Fundamentals of Optics, 1959.
9. Kane, J. A., J. Atmos. Terr. Phys. 23, 338, 1961.
10. Cook, C. J. and Lorents, D. C., Stanford Research Institute project, #PAU-3340, Final Tech. Reprot #6, 1961.
11. Molmud, P., Phys. Rev. 114, 29, 1959.
12. Allis, W. P., Handbuch der Physik, 21, 332, 1956.
13. Shkarofsky, I. P., Can J. Phys. 39, 1619, 1961.
14. Mott, N. F. and Massey, H. S. W., The Theory of Atomic Collisions, Oxford, 1949.
15. Temkin, A., Phys. Rev. 107, 1004, 1957.
16. Bates, D. R. and Massey, H. S. W., Proc. Roy. Soc. (London), A192, 1, 1947.

17. Johnson, F. S., Satellite Environment Handbook, Lockheed Missles and Space Division, LMSD-895006, 1960.

18. Naynaber, R. H. and Marino, L. L., Rothe, E. W. and Trujillo, S. M., Phys. Rev. 123, 148, 1961.

19. Pack, J. L. and Phelps, A. V., Phys. Rev. 121, 798, 1961.

20. Phelps, A. V., Fundingsland, O. T., and Brown, S. C., Phys. Rev. 84, 559, 1951.

21. Huxley, L. G. H., J. Atmos. and Terr. Phys. 16, 46, 1959.

22. Phelps, A. V., J. Appl. Phys. 31, 1723, 1960.

23. Van Lint, Report #TR59-43, General Dynamics Corporation (1959).

24. Kelly, D. C., Phys. Rev. 119, 27, 1960.

25. Kallman-Bijl, H. K., Space Research II, Proceedings of the Second International Space Sciences Symposium, 889, 1961.

26. Bauer, S. J., private communication.

27. Dingle, R. B., Arndt, D. and Roy, S. K., Appl. Sci. Res. B6, 155, 245, 1957.

28. Appleton, E. V., Piggott, W. R., Proc. Gen. Assy. V.R.S.I. Stockholm, 320, 1948.

29. Fang, P. H., Applied Sci. Research B9, 51, 1961.

30. Chandra, S., NASA Technical Report X-615-62-90, 1962.

31. Machin, K. E., Ryle, M. and Vonberg, D. D., Proc. I.E.E. 99, 127, 1952.

32. Goldman, S., Frequency Analysis, Modulation, and Noise, 223, 1948.

33. Shklovsky, I. S., Cosmic Radio Waves, 54, 1960.

34. Nassau, J. J., Practical Astronomy, 23, 1932.

35. The American Ephemeris and Nautical Almanac, U. S. Printing Office (published for each year).

36. Ratcliffe, J. A., Magneto-ionic Theory, 109, 1959.

37. Reber, G. and Ellis, G. R., J. Geophys. Res. 61, 1, 1956.

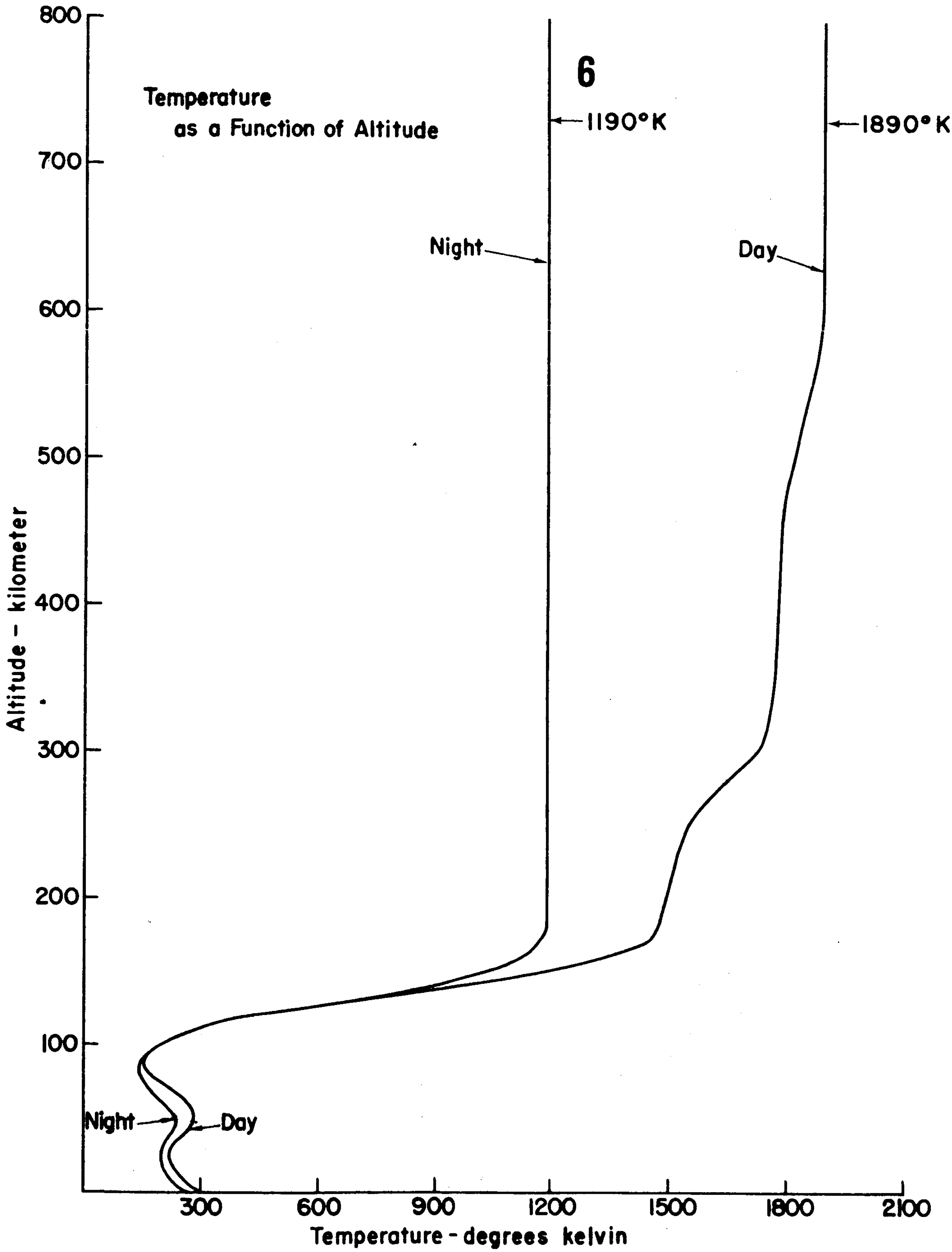

6
Temperature
as a Function of Altitude
1190°K
1890°K
Night
Day
Night
Day
800
700
600
500
400
300
200
100
300
600
900
1200
1500
1800
2100
Altitude - kilometer
Temperature - degrees kelvin

Figure 1

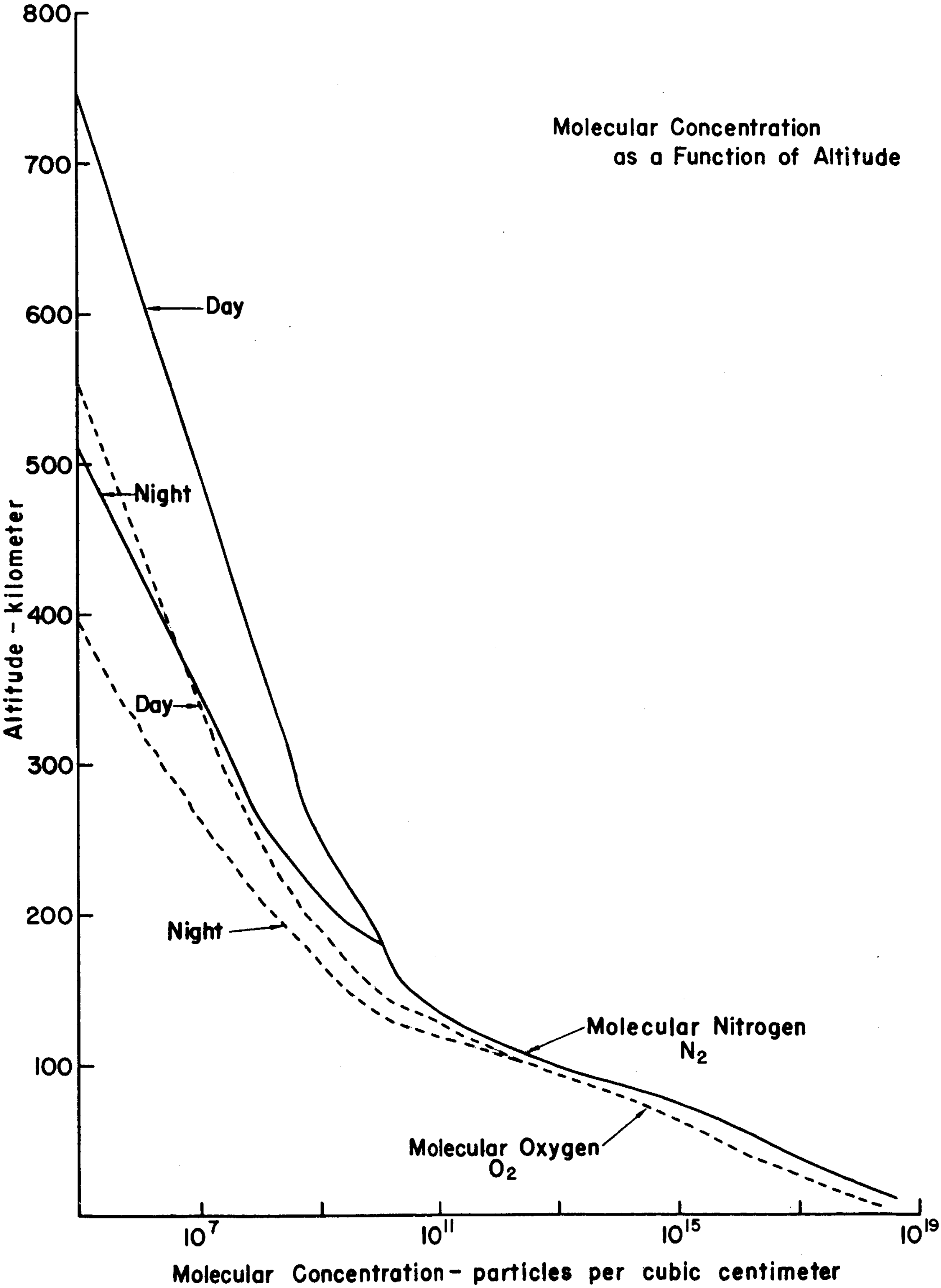
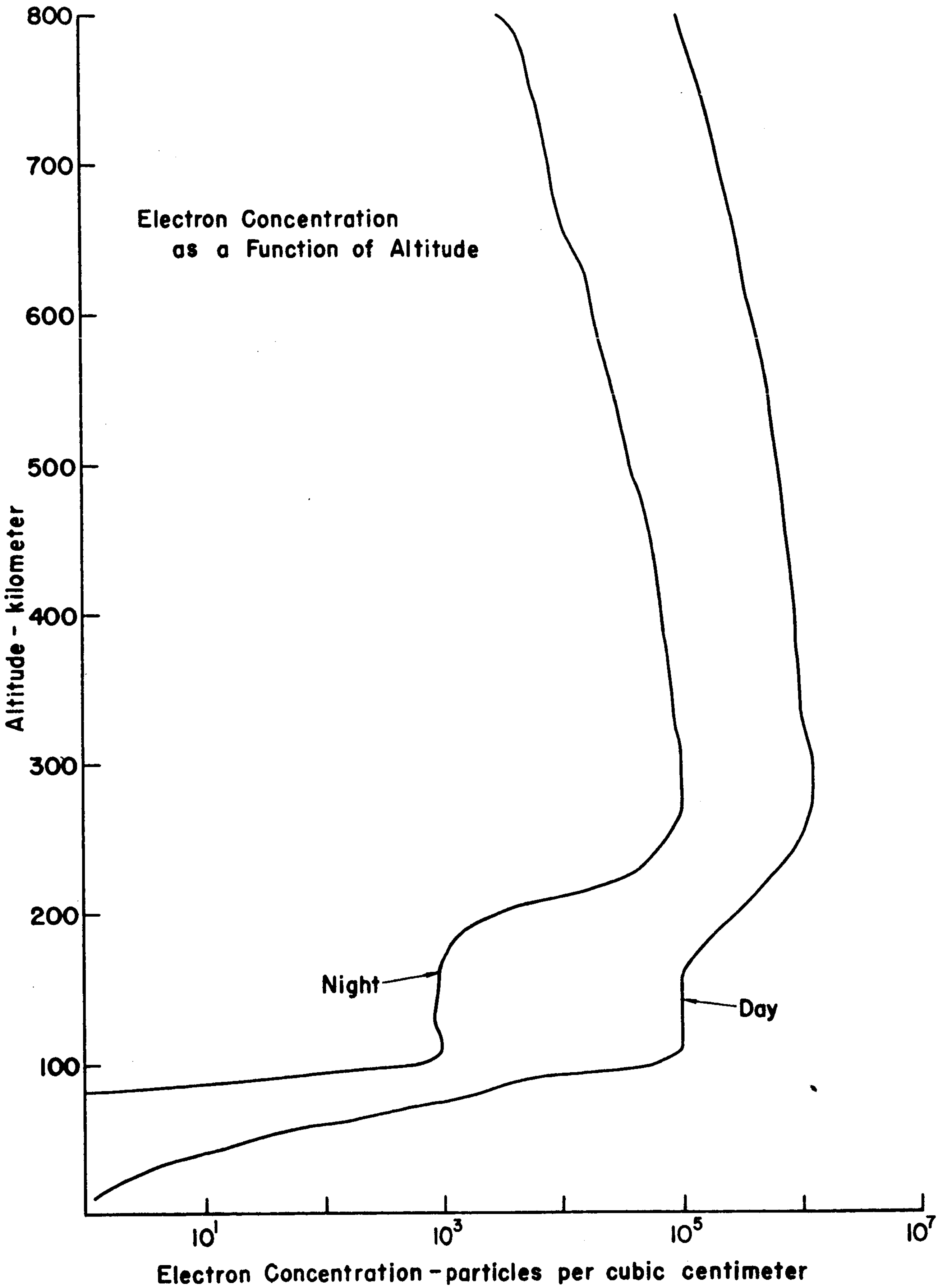

Molecular Concentration
as a Function of Altitude
Altitude - kilometer
800
700
600
500
400
300
200
100
Day
Night
Day
Night
Molecular Nitrogen
N_2
Molecular Oxygen
O_2
10^7
10^{11}
10^{15}
10^{19}
Molecular Concentration - particles per cubic centimeter

Figure 2



800
700
600
500
400
300
200
100
Altitude - kilometer
Electron Concentration
as a Function of Altitude
Night
Day
10^1
10^3
10^5
10^7
Electron Concentration - particles per cubic centimeter

Figure 3

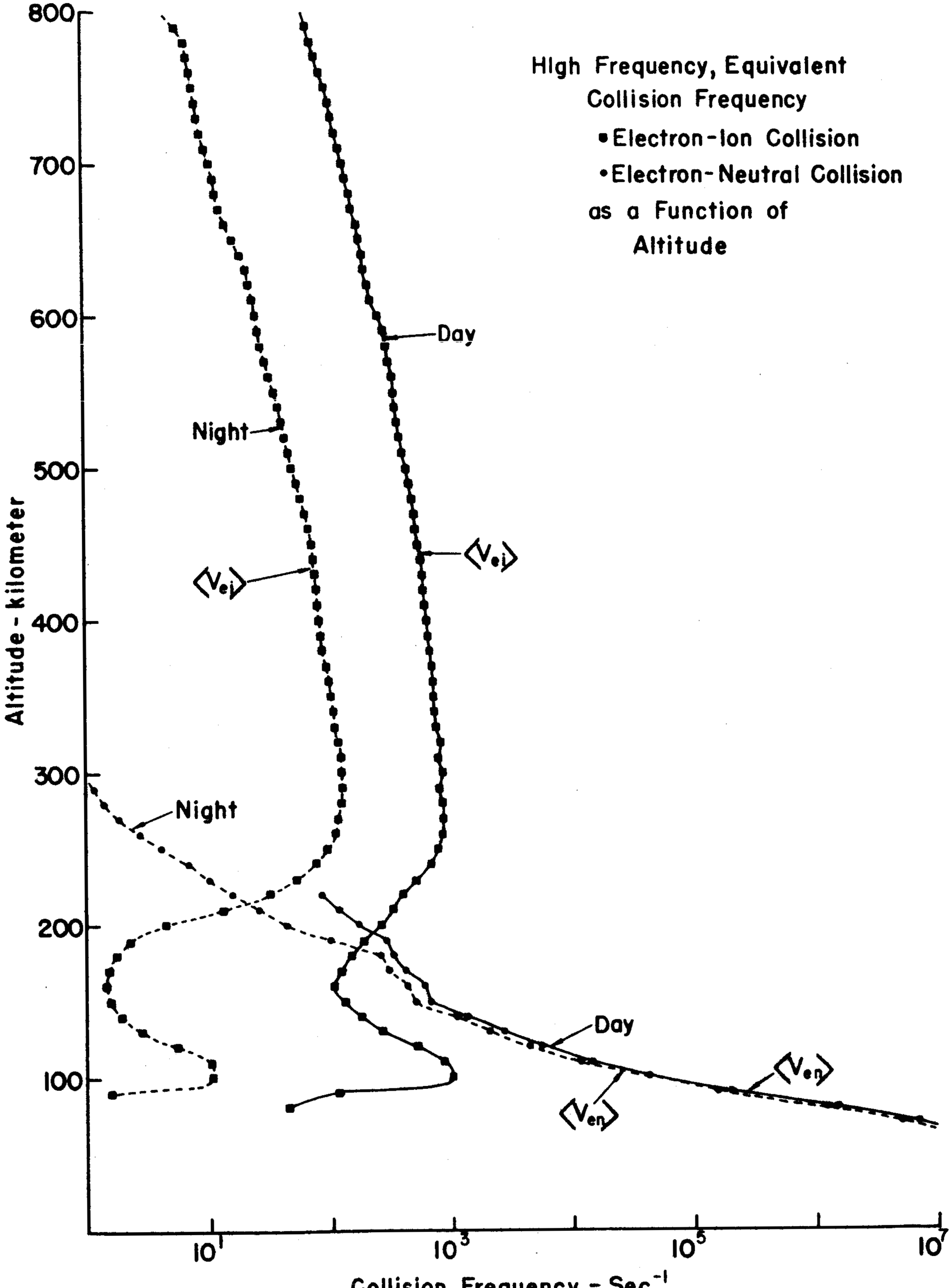

High Frequency, Equivalent
Collision Frequency
■ Electron-Ion Collision
• Electron-Neutral Collision
as a Function of
Altitude
800
700
600
500
400
300
200
100
Altitude - kilometer
Day
Night
⟨V_{ei}⟩
⟨V_{ei}⟩
Night
Day
⟨V_{en}⟩
⟨V_{en}⟩
10^1
10^3
10^5
10^7
Collision Frequency - Sec^{-1}

Figure 4

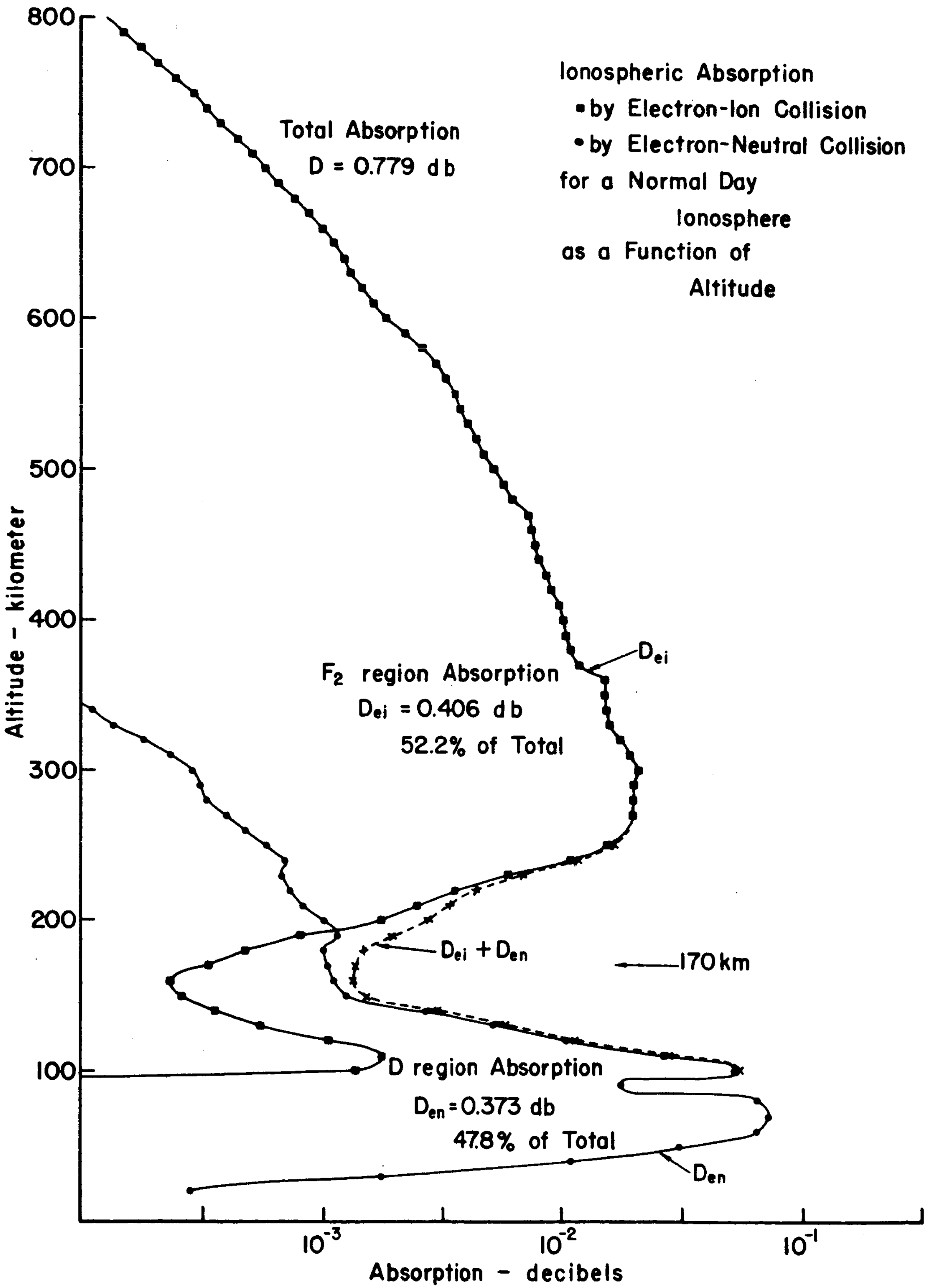
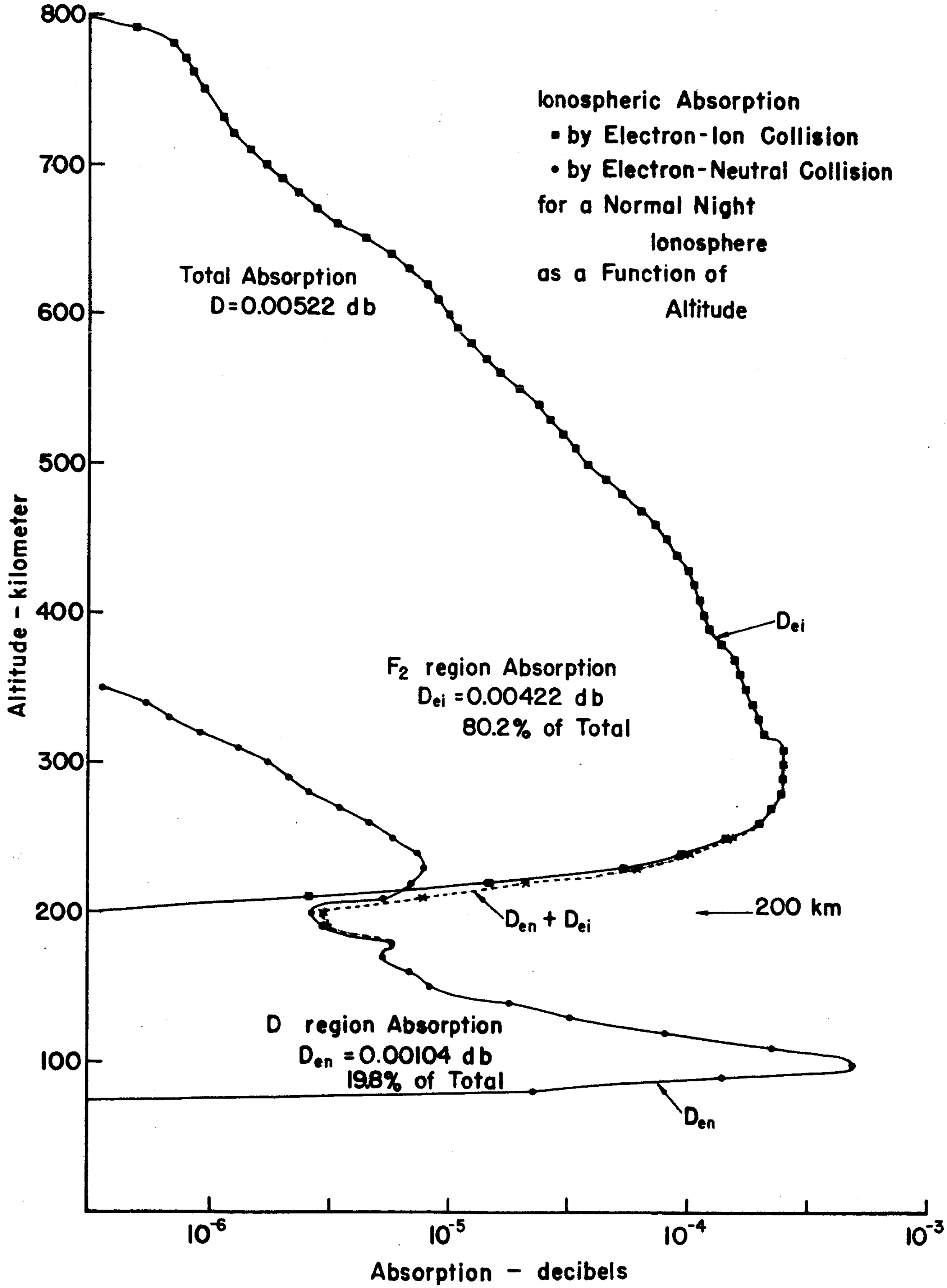

Ionospheric Absorption
• by Electron-Ion Collision
• by Electron-Neutral Collision
for a Normal Day
Ionosphere
as a Function of
Altitude
Total Absorption
D = 0.779 db
F2 region Absorption
Dei = 0.406 db
52.2% of Total
Dei
Dei + Den
170 km
D region Absorption
Den = 0.373 db
47.8% of Total
Den
Altitude - kilometer
800
700
600
500
400
300
200
100
10-3
10-2
10-1
Absorption - decibels

Figure 5



Ionospheric Absorption
▪ by Electron-Ion Collision
• by Electron-Neutral Collision
for a Normal Night
Ionosphere
as a Function of
Altitude
Total Absorption
D=0.00522 db
F2 region Absorption
Dei =0.00422 db
80.2% of Total
D region Absorption
Den =0.00104 db
19.8% of Total
Dei
Den + Dei
Den
200 km
800
700
600
500
400
300
200
100
Altitude - kilometer
10-6
10-5
10-4
10-3
Absorption - decibels

Figure 6

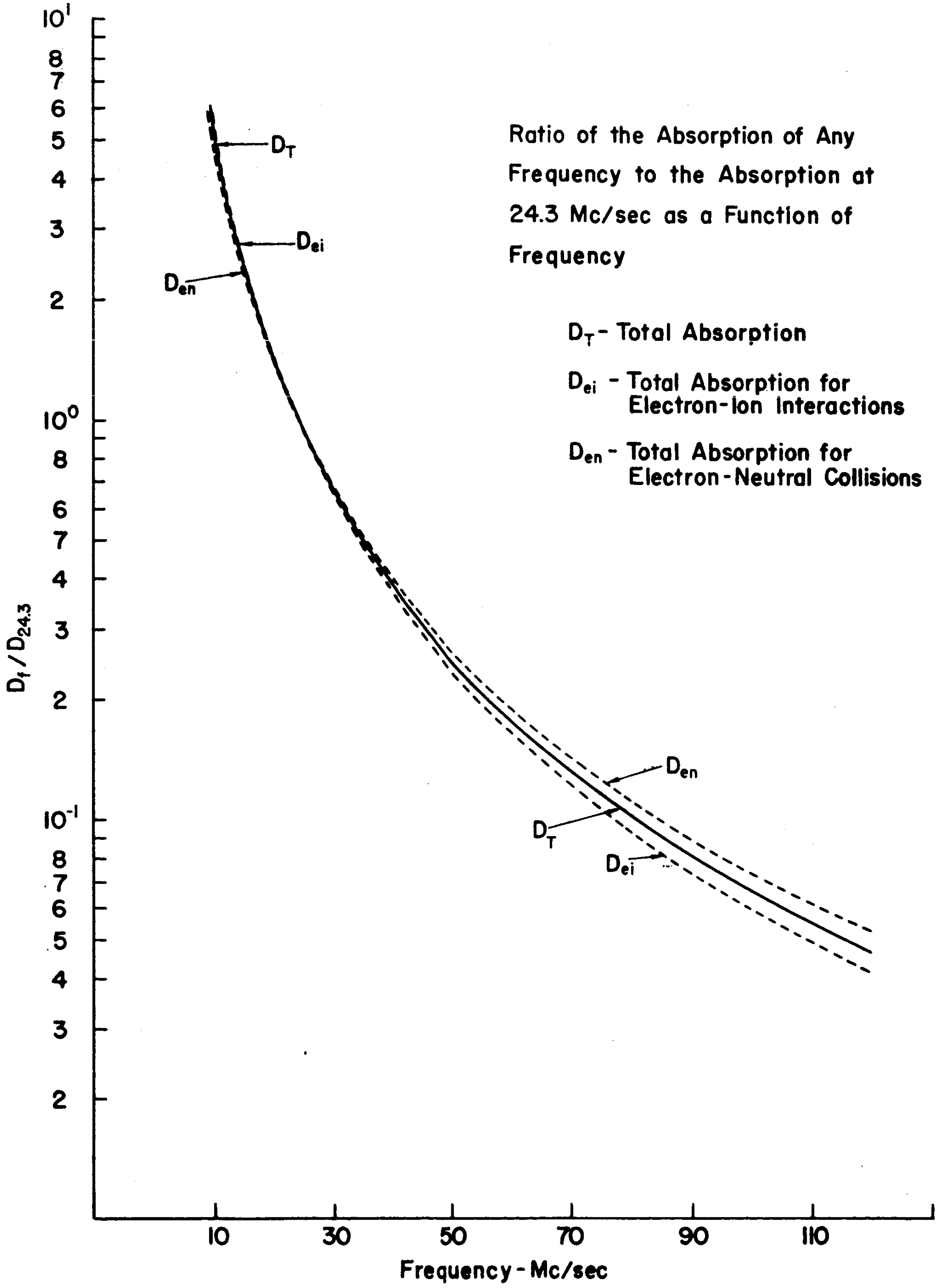
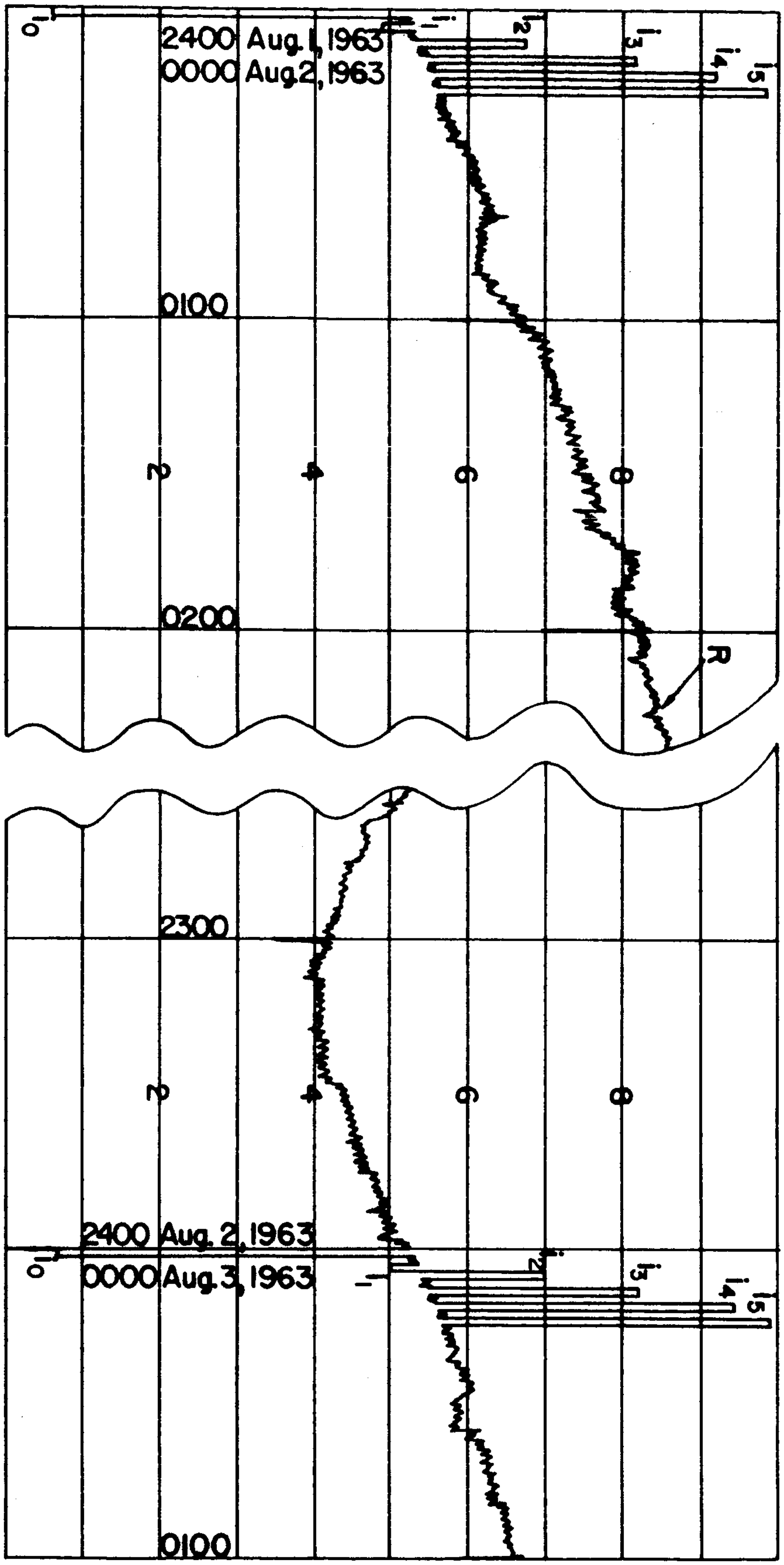
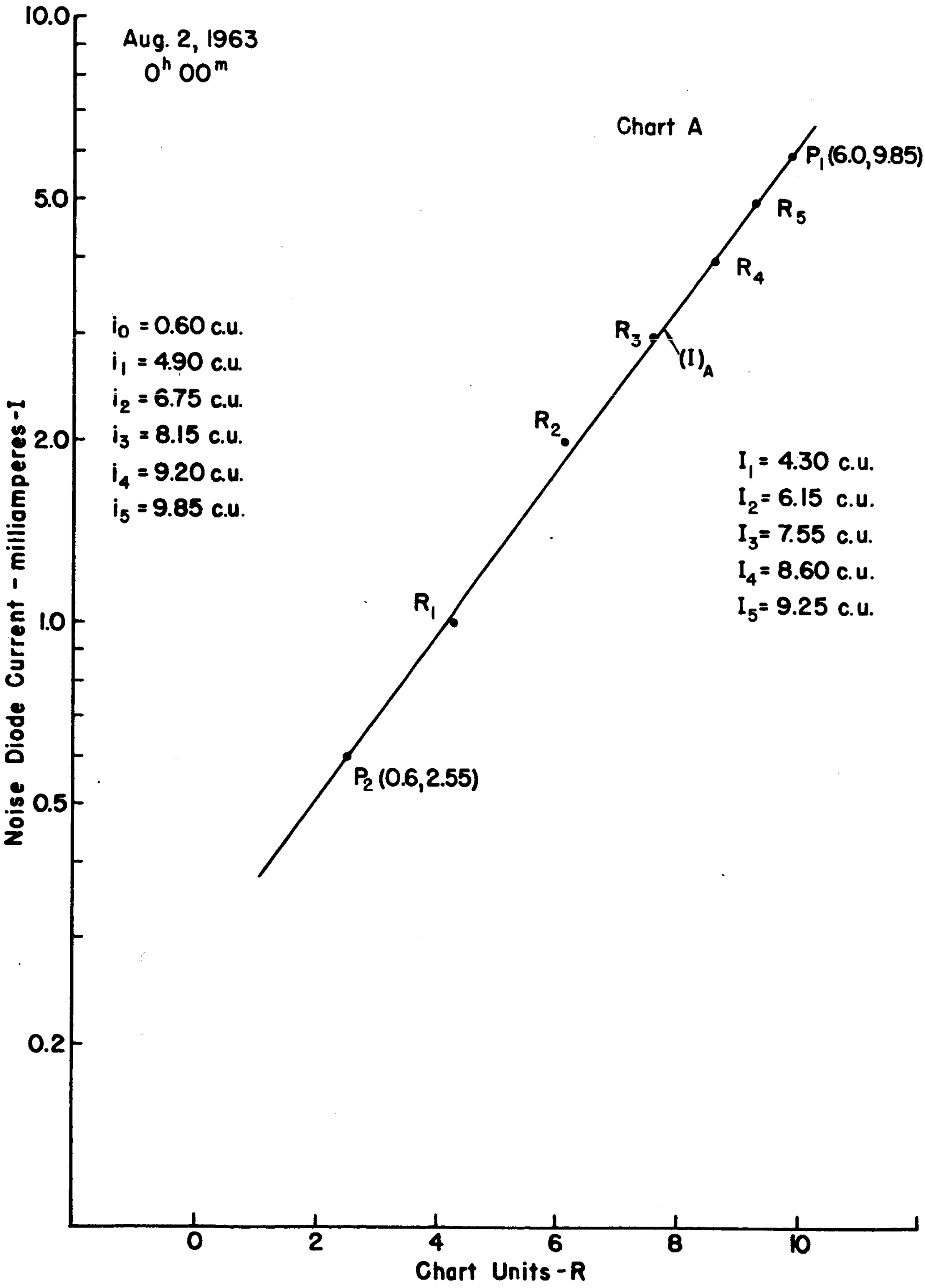

Ratio of the Absorption of Any Frequency to the Absorption at 24.3 Mc/sec as a Function of Frequency
D_T - Total Absorption
D_{ei} - Total Absorption for Electron-Ion Interactions
D_{en} - Total Absorption for Electron-Neutral Collisions
D_T
D_{ei}
D_{en}
$D_f / D_{24.3}$
10^1
10^0
10^{-1}
Frequency - Mc/sec
10
30
50
70
90
110

Figure 7



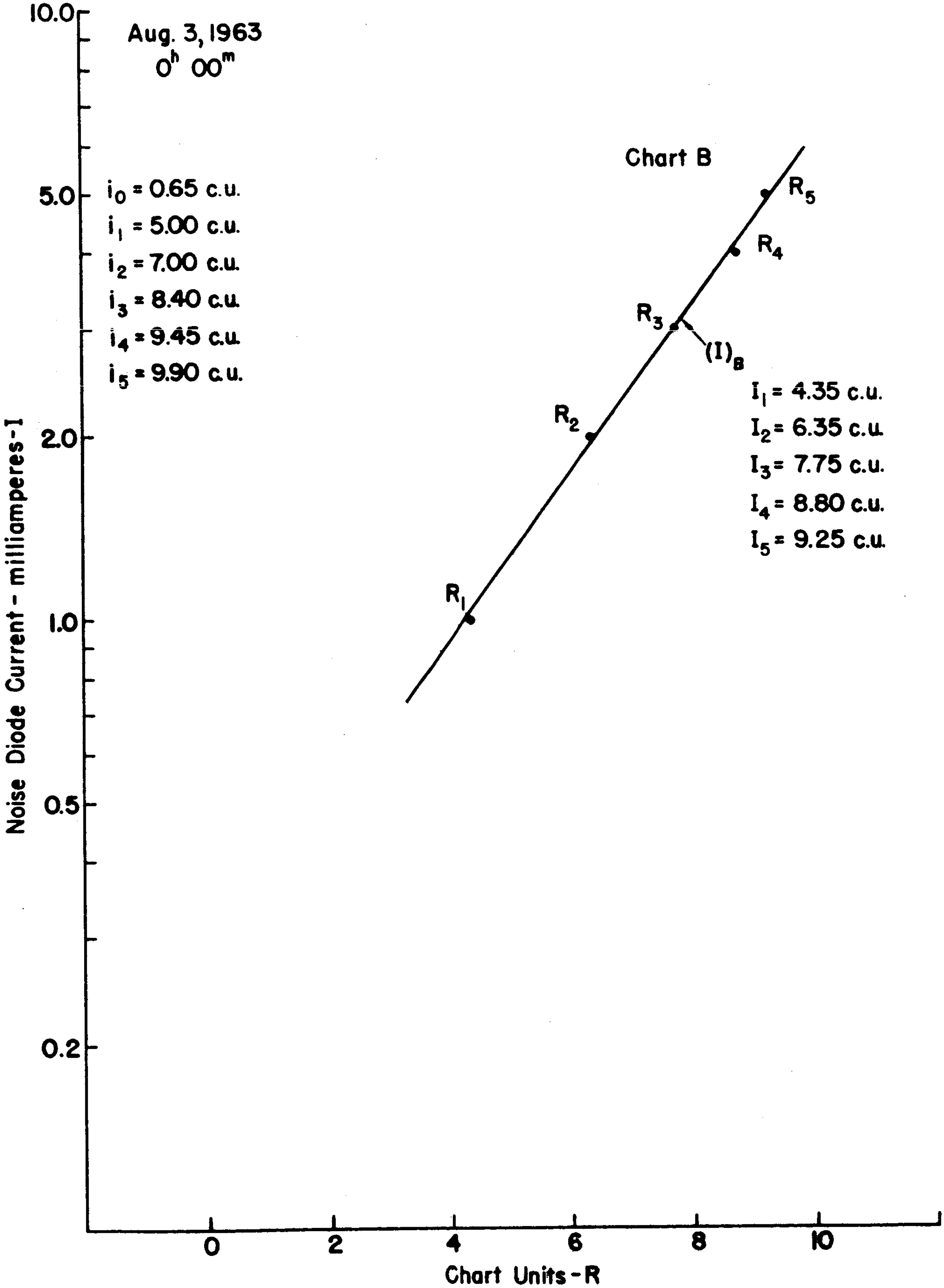
Typical Riometer Record

Figure 8



Aug. 2, 1963
$0^h\ 00^m$
Chart A
P_1 (6.0, 9.85)
R_5
R_4
R_3
$(I)_A$
R_2
R_1
P_2 (0.6, 2.55)
i_0 = 0.60 c.u.
i_1 = 4.90 c.u.
i_2 = 6.75 c.u.
i_3 = 8.15 c.u.
i_4 = 9.20 c.u.
i_5 = 9.85 c.u.
I_1 = 4.30 c.u.
I_2 = 6.15 c.u.
I_3 = 7.55 c.u.
I_4 = 8.60 c.u.
I_5 = 9.25 c.u.
Noise Diode Current - milliamperes - I
10.0
5.0
2.0
1.0
0.5
0.2
0
2
4
6
8
10
Chart Units - R

Figure 9



Aug. 3, 1963
0h 00m
Chart B
i_0 = 0.65 c.u.
i_1 = 5.00 c.u.
i_2 = 7.00 c.u.
i_3 = 8.40 c.u.
i_4 = 9.45 c.u.
i_5 = 9.90 c.u.
R_1
R_2
R_3
R_4
R_5
$(I)_B$
I_1 = 4.35 c.u.
I_2 = 6.35 c.u.
I_3 = 7.75 c.u.
I_4 = 8.80 c.u.
I_5 = 9.25 c.u.
10.0
5.0
2.0
1.0
0.5
0.2
0
2
4
6
8
10
Noise Diode Current - milliamperes-I
Chart Units-R

Figure 10